

# 拡張位相グラフ(EPG)の基礎

寺田康彦

筑波大学 数理物質系 物理工学域

<http://www.bk.tsukuba.ac.jp/~mrlab/index.html>

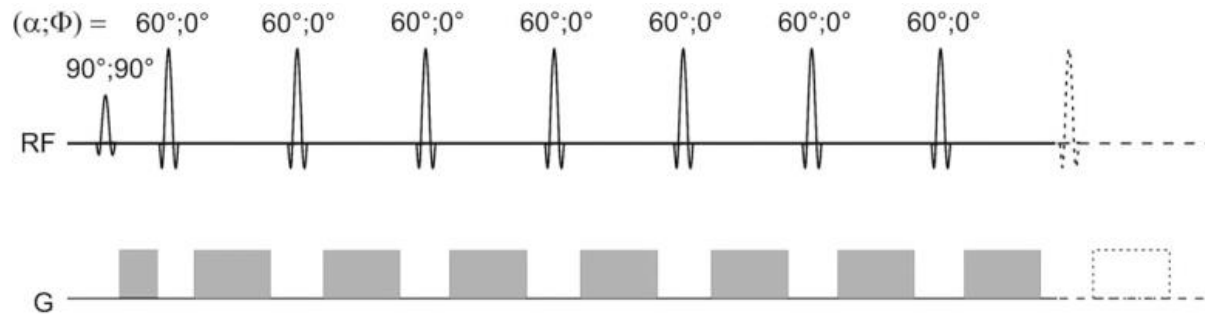
2018/9/7 第46回日本磁気共鳴医学会大会  
教育講演基礎 1

# EPGの特徴

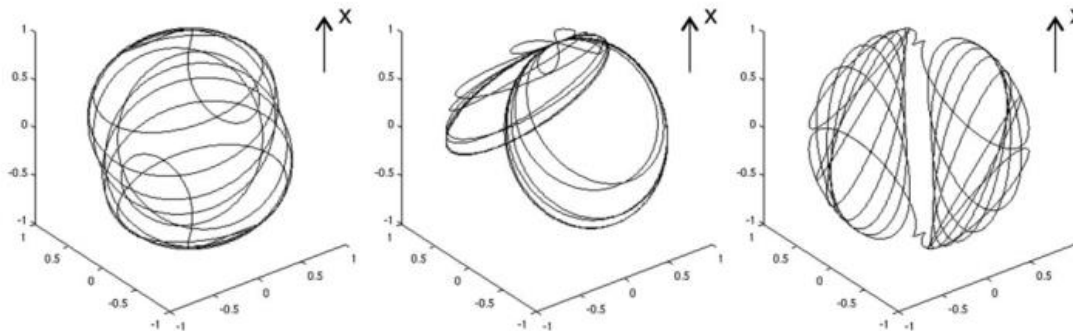
1. EPG（拡張位相グラフ）は1ボクセル・シミュレーションである
2. マルチエコーの強度を簡単に計算できる
3. エコーの起源を理解できる
4. 視覚的にも理解しやすい

参考文献 [1] **M. Weigel**, Extended Phase Graphs: Dephasing, RF Pulses, and Echoes - Pure and Simple, JMRI, 41:266-295 (2015)

# なぜEPGを使うのか？



Magnetization distributions of the CPMG sequence from unknown time points :



Bloch  
simulation

Weigel論文より  
引用

- 「エコー」はどこに現れる？
- 「エコー」の種類は？ スピンエコー？ STE？
- 「エコー」の強度は？

# EPGとは？

RFパルスの働き

古典的位相グラフ Phase graph (PG)



勾配磁場 (dephasing, rephasing)

核磁化分布のフーリエ表現

緩和

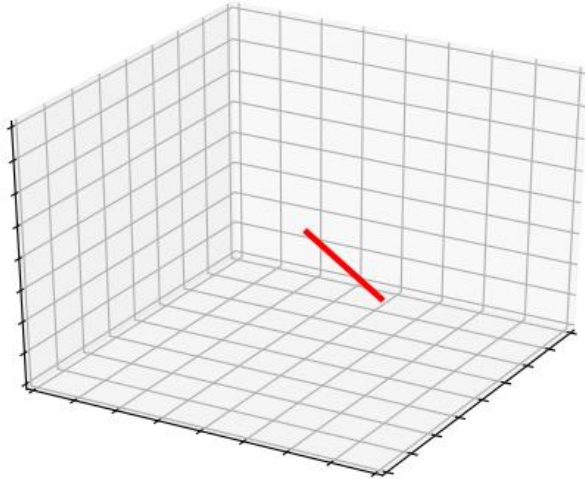
拡散・流れなど



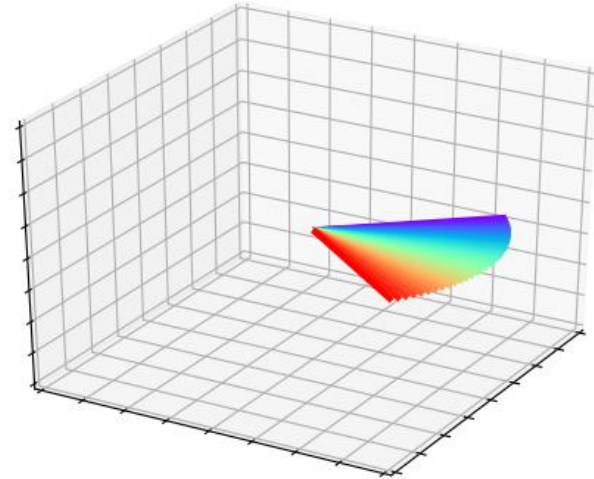
拡張位相グラフ Extended phase graph (EPG)

# 用語の整理：ISOCHROMATとは？

spin isochromat



spin isochromats



同じ位相をもつスピンの  
集合体

spin isochromatの集合体

便宜上「**単一スピン**」  
と呼ぶことにする

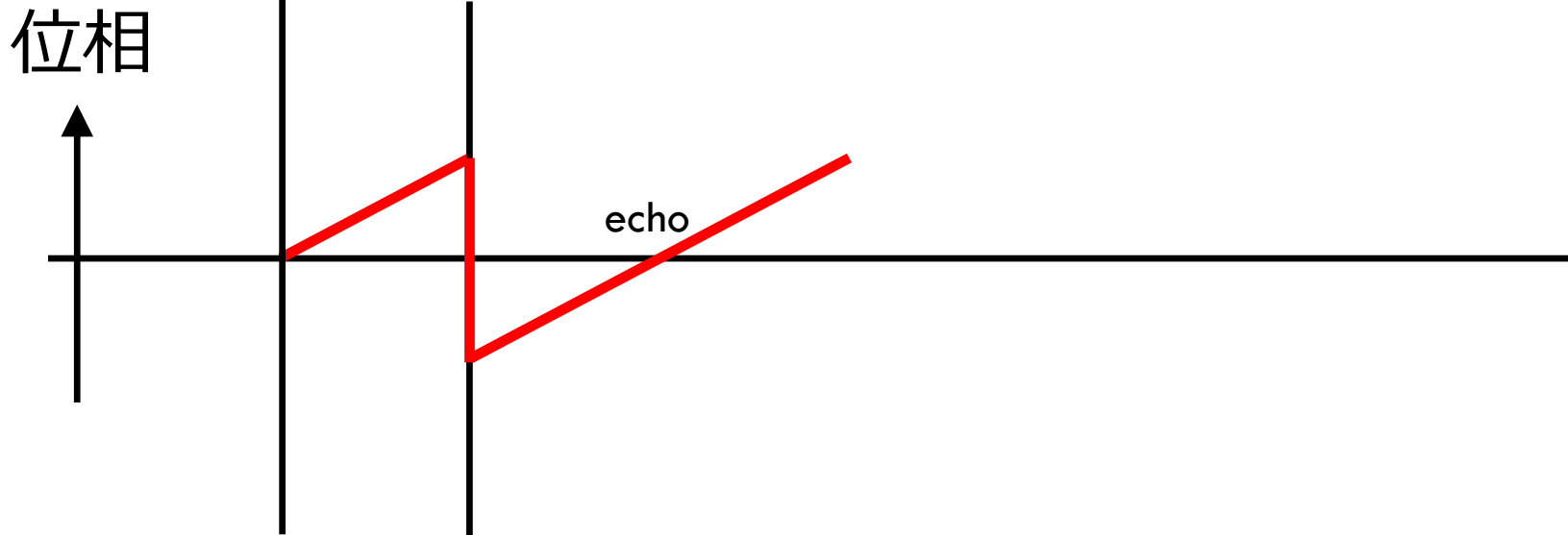
便宜上「**多数スピン**」  
と呼ぶことにする

※本当は「単一」ではないことに注意

# 古典的な位相グラフ

## スピンエコー

$90^\circ_y$      $180^\circ_x$



### 暗黙の了解

1. ボクセル内には**多数のスピン**が存在
2. ボクセル内は**磁場が不均一**  
(線形近似=background gradient)



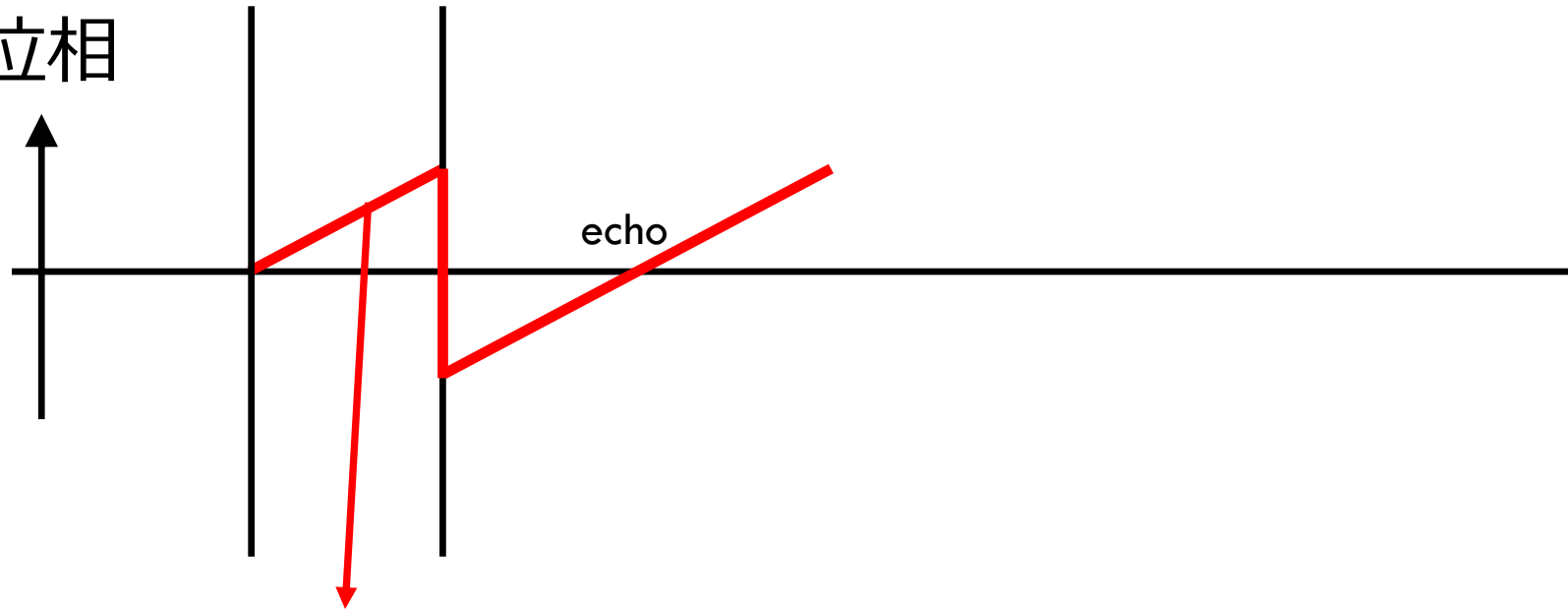
「エコー」という現象を考えるのに必須

# 古典的な位相グラフ

## スピンエコー

$90^\circ_y$     $180^\circ_x$

位相



この線は何？

多数あるスピンのうち

ある単一スピン (spin isochromat) に注目した時の  
横磁化の位相の時間変化を表す

# EPGとは？

RFパルスの働き

古典的位相グラフ Phase graph (PG)



勾配磁場 (dephasing, rephasing)

? ? ?

核磁化分布のフーリエ表現

緩和

拡散・流れなど



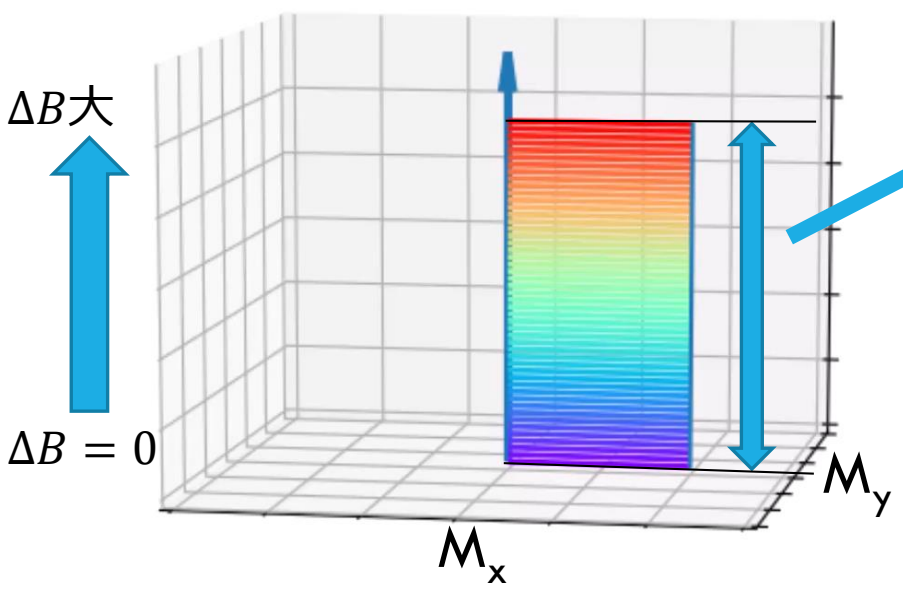
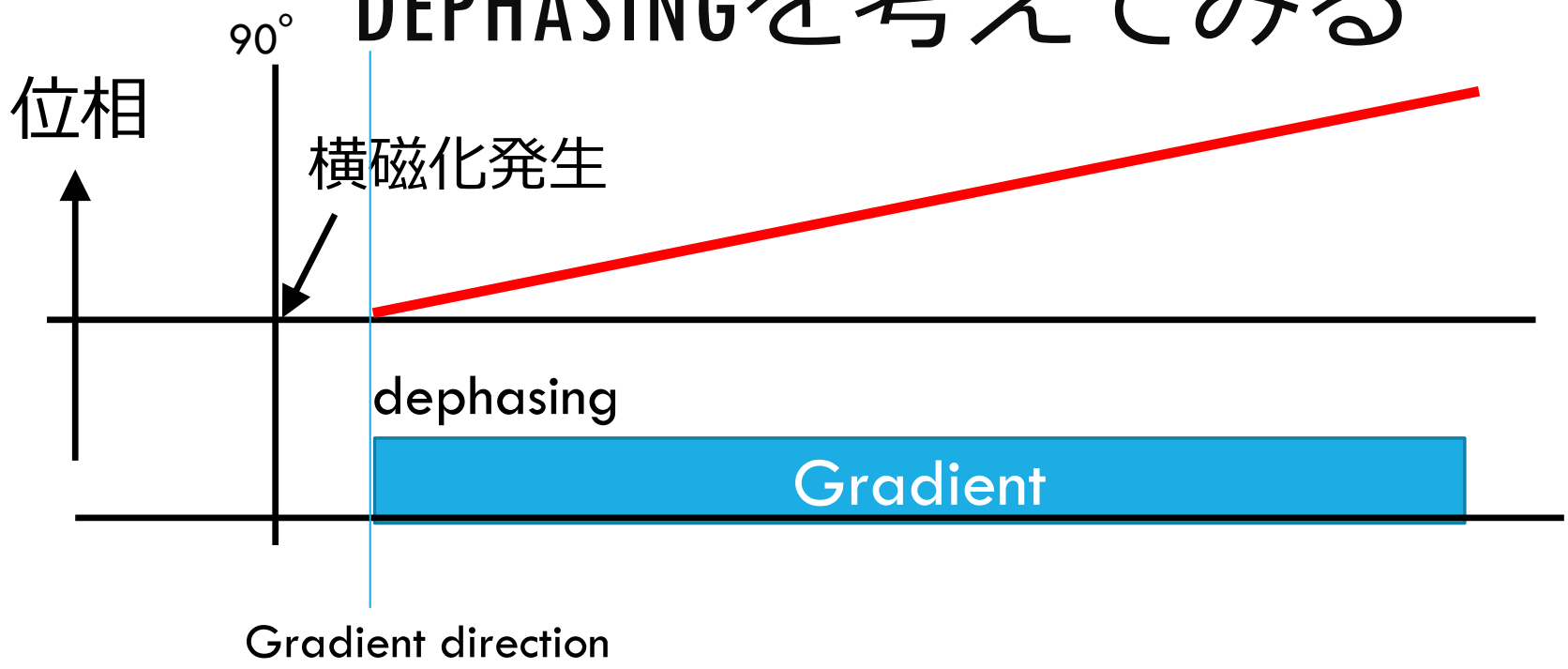
拡張位相グラフ Extended phase graph (EPG)



# EPGとは？

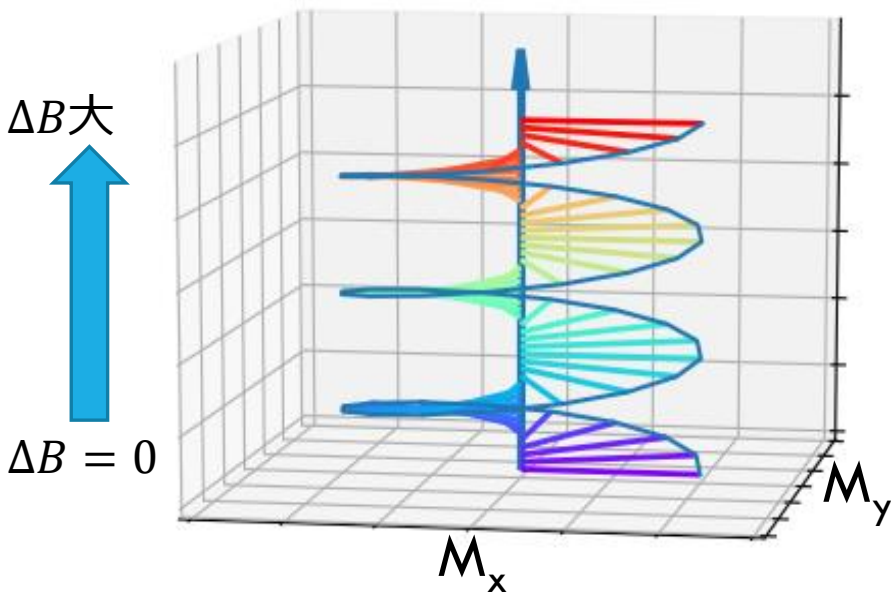
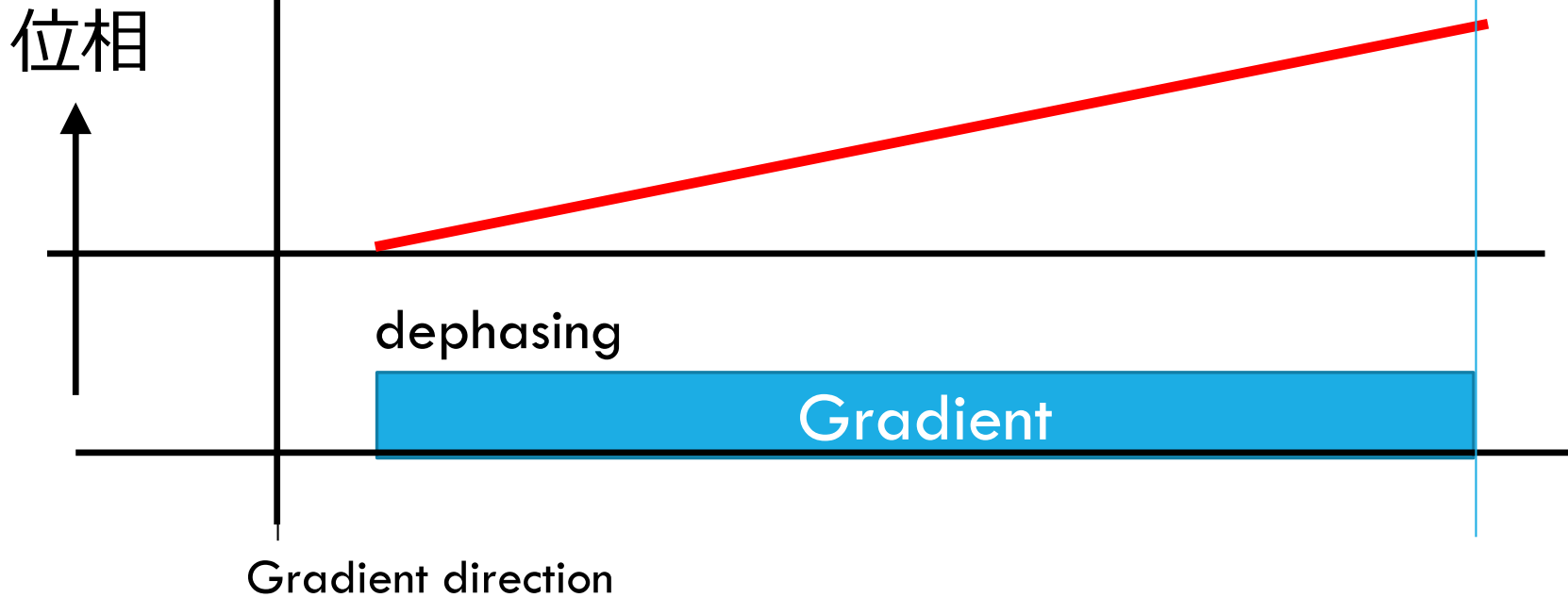
まず横磁化を考える

# DEPHASINGをを考えてみる



EPGでは、  
1ボクセル内の多数  
スピンを考える

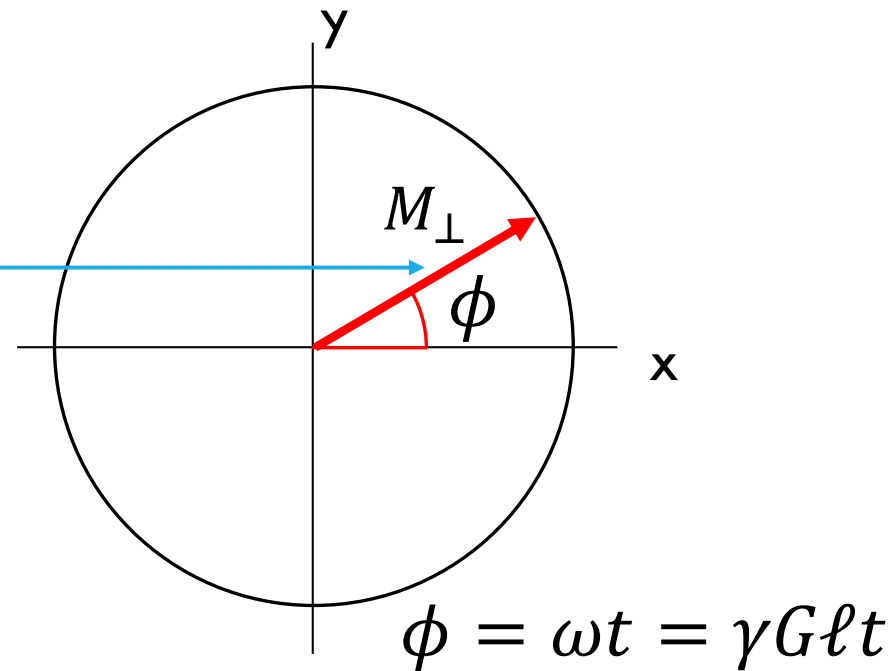
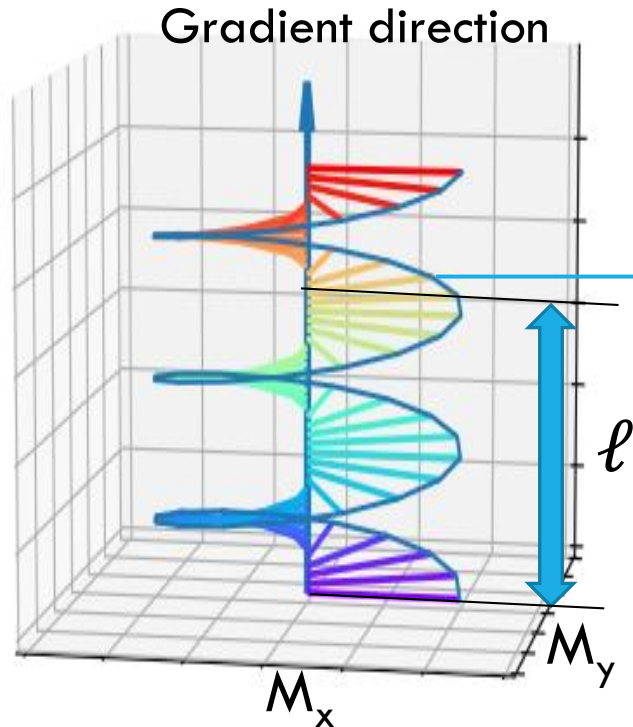
# なぜフーリエ変換なのか？



- ・ 横磁化の位相が変化して、螺旋が捻じれていく
- ・ 螺旋のねじれ具合 = dephasingの程度

定量的に表現したい

# kの導入



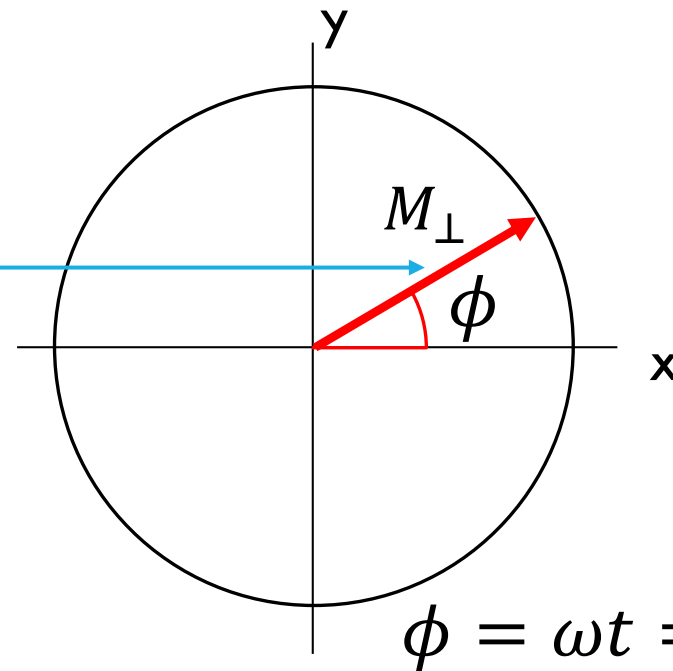
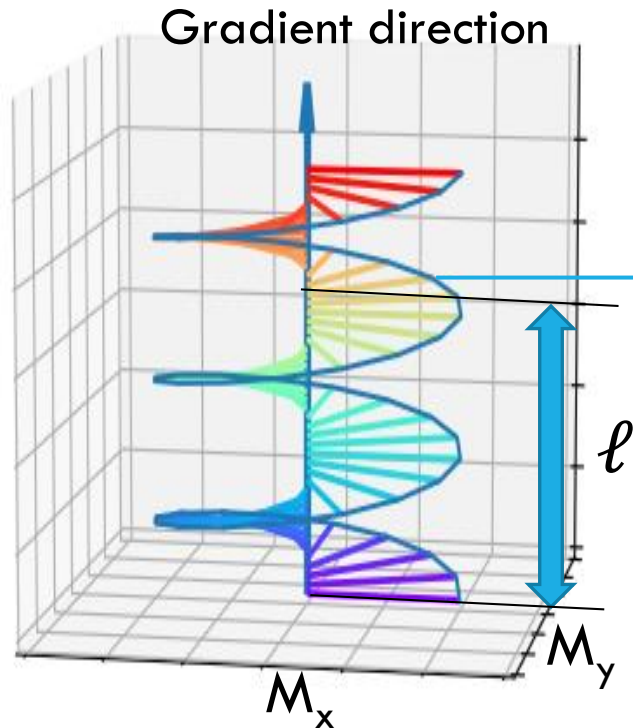
$k = \gamma \int G(t') dt'$  で定義すると

$$\phi = k l$$

$k$ は螺旋のねじれ具合、つまり dephasing の程度を表す



# kの導入



$$\phi = \omega t = \gamma G \ell t = k \ell$$



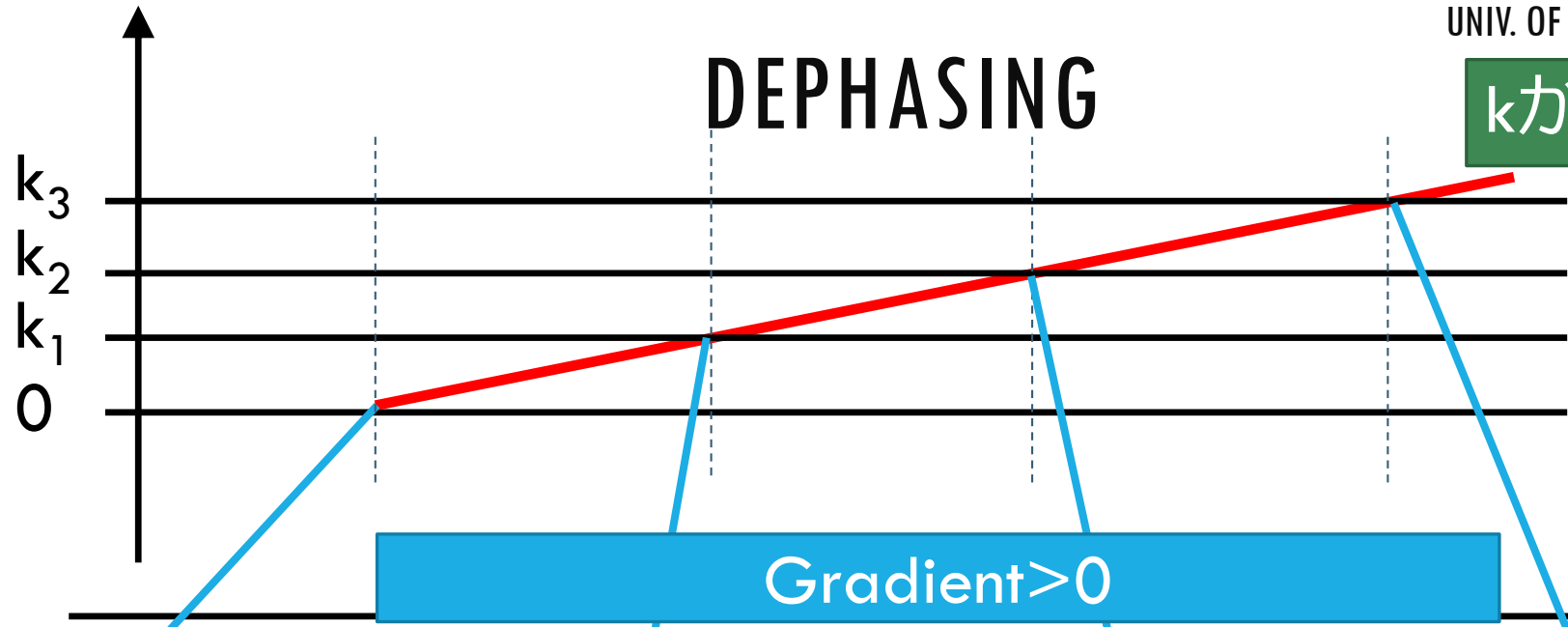
$$\begin{cases} M_x = M_{\perp} \cos(k\ell) & \text{横磁化のx成分} \\ M_y = M_{\perp} \sin(k\ell) & \text{横磁化のy成分} \end{cases}$$

$$M_+(\ell) = M_x + iM_y = M_{\perp} e^{ik\ell}$$

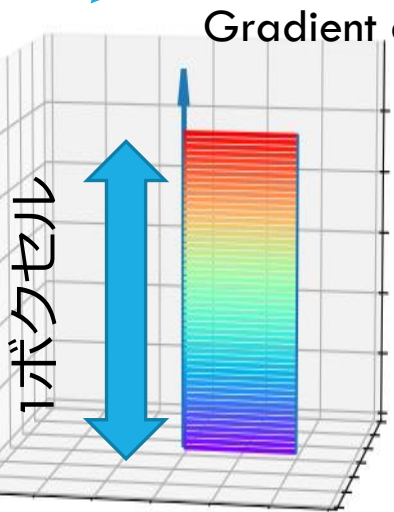
複素表現

# DEPHASING

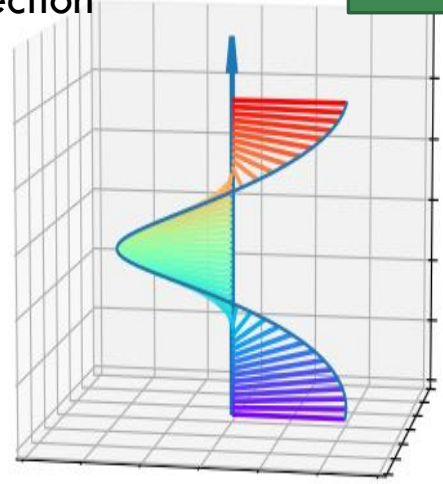
**kが増加**



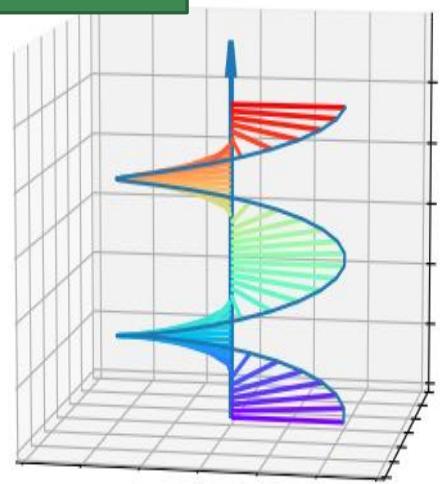
**右ねじ螺旋**



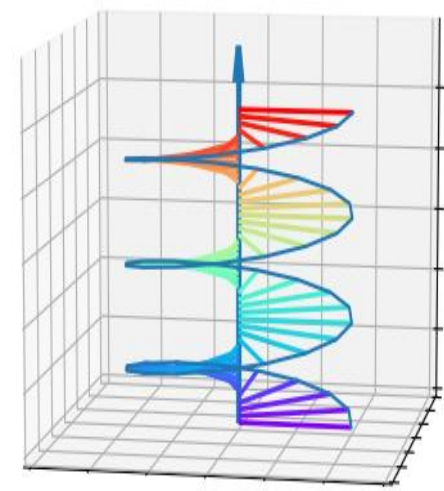
0



$2\pi/\text{voxel}$



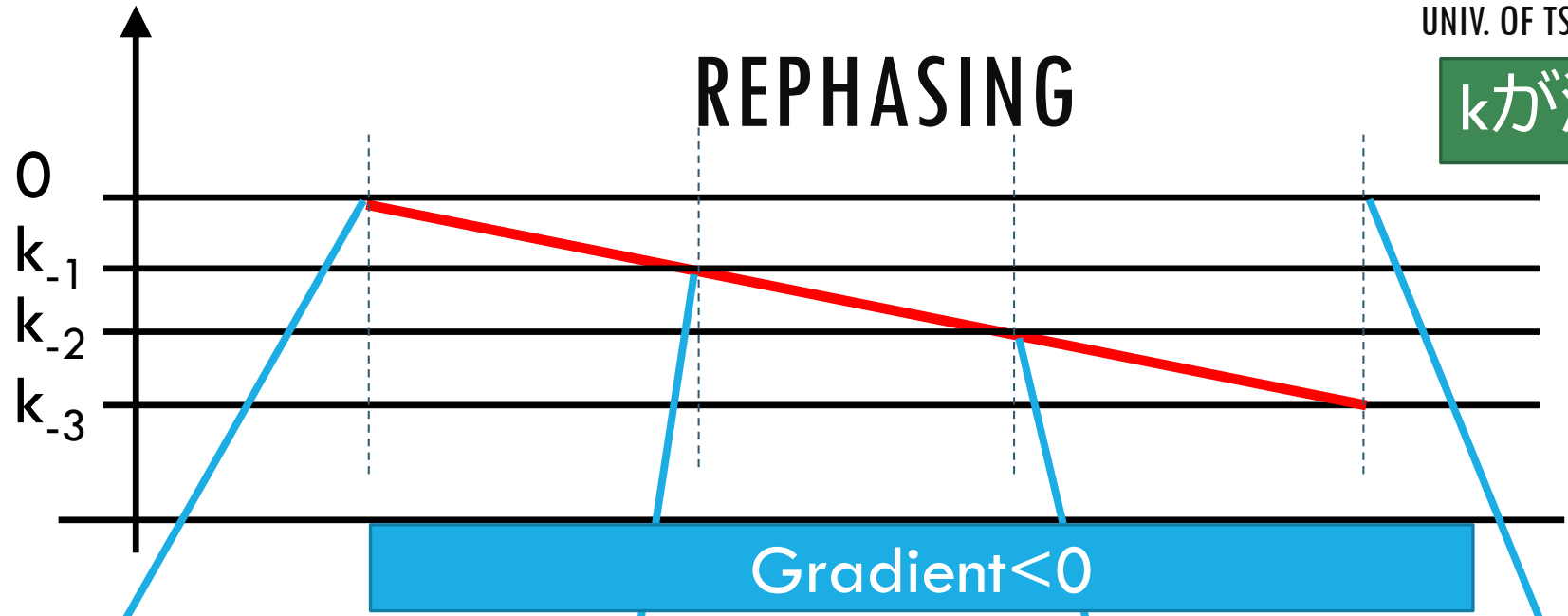
$4\pi/\text{voxel}$



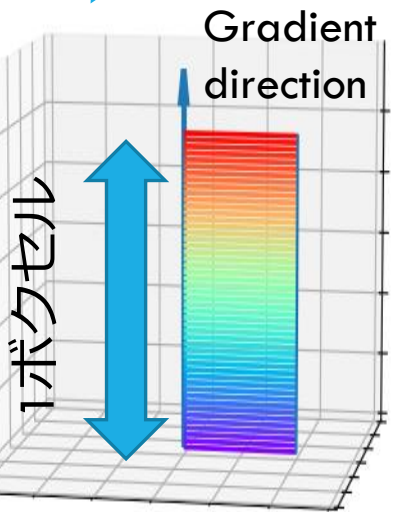
$6\pi/\text{voxel}$

# REPHASING

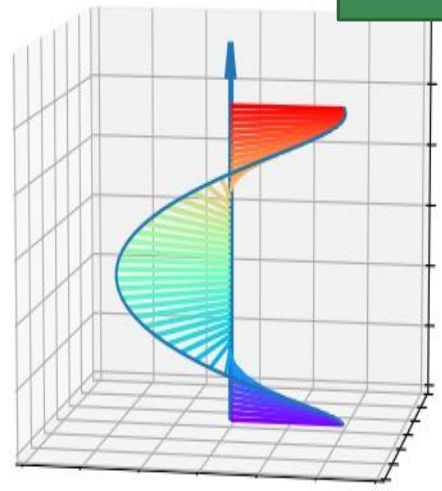
**kが減少**



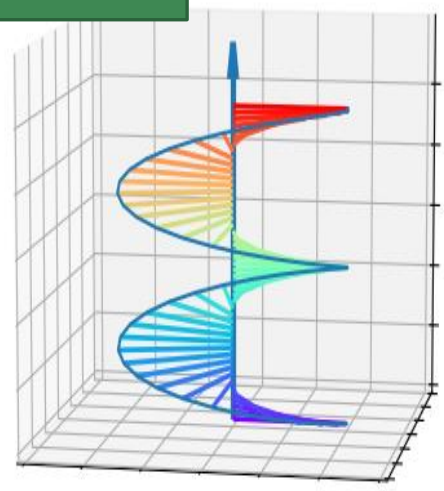
**左ねじ螺旋**



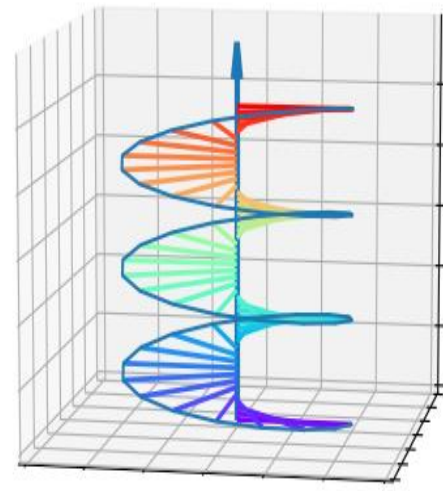
0



$-2\pi/\text{voxel}$

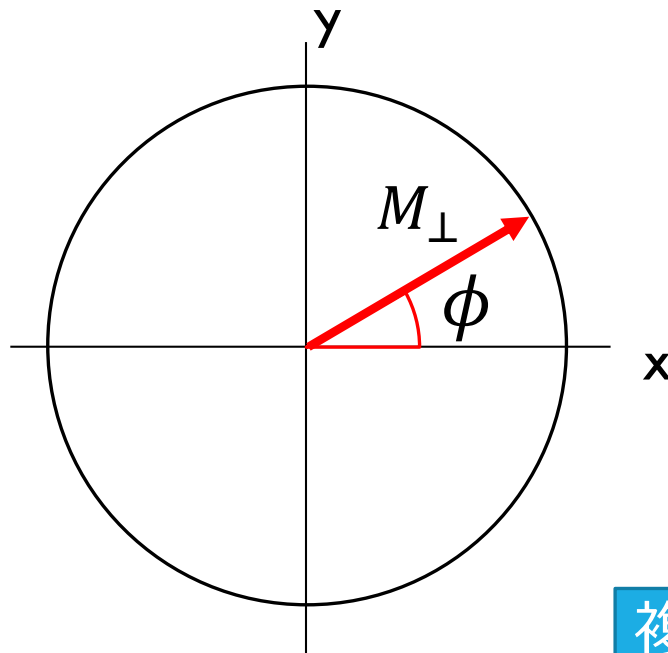


$-4\pi/\text{voxel}$

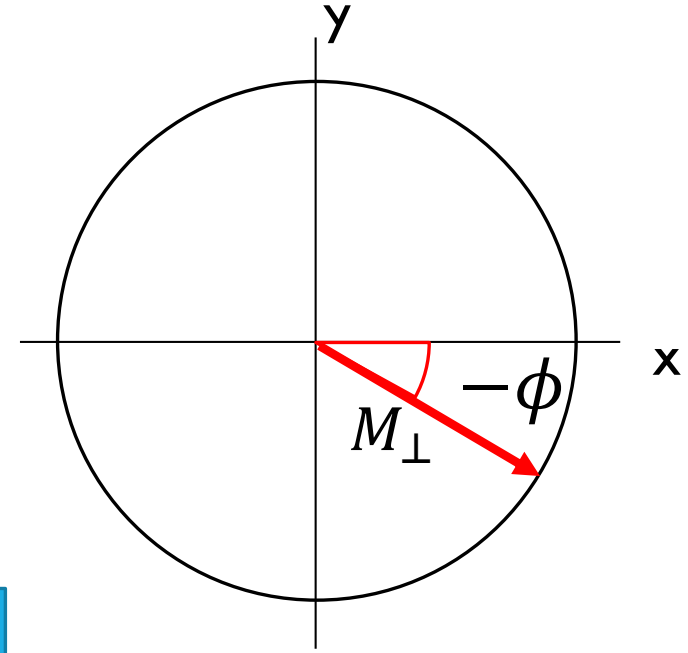


$-6\pi/\text{voxel}$

# 左ねじ螺旋をどう表現する？



複素共役



$$M_{+}(\ell) = M_x + iM_y = M_{\perp}e^{ik\ell} \longleftrightarrow M_{-}(\ell) = M_x - iM_y = M_{\perp}e^{-ik\ell}$$

$k$ が正：右ねじ  
 $k$ が負：左ねじ

$k$ が正：左ねじ  
 $k$ が負：右ねじ

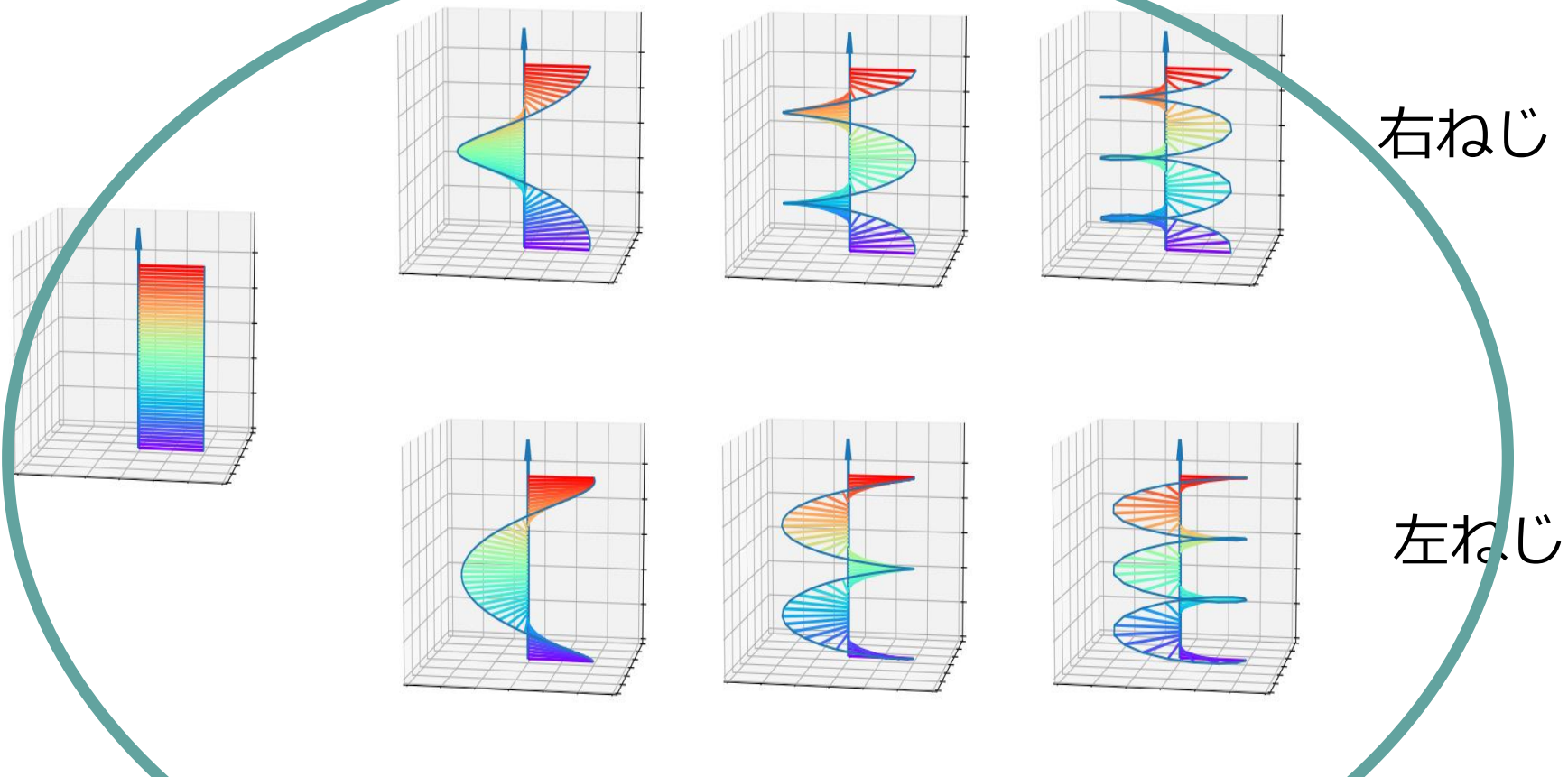
“dephasing magnetization”

“rephasing magnetization”

表現法に冗長性がある



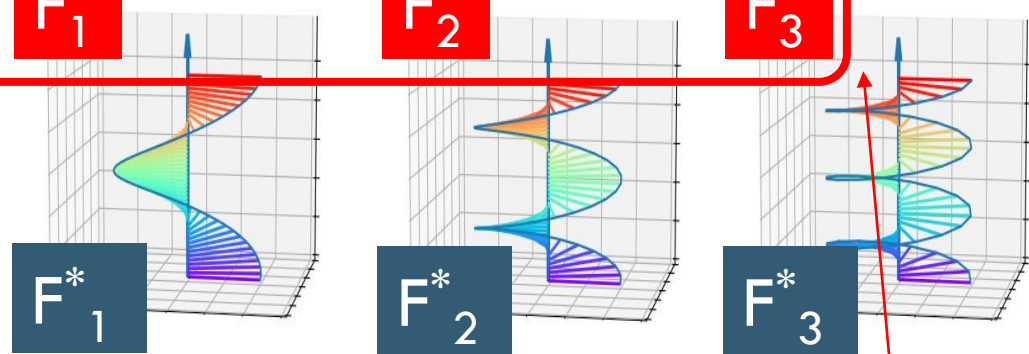
# フーリエ変換：多数の螺旋の重ね合わせ



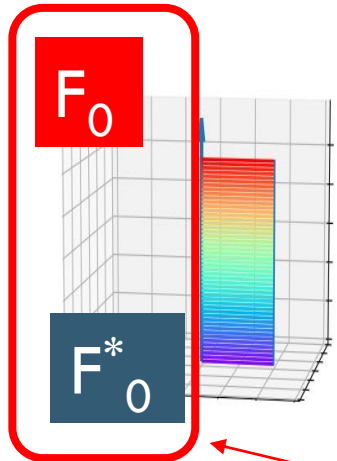
任意の横磁化分布は  
すべての螺旋の重ね合わせ

# フーリエ変換：多数の螺旋の重ね合わせ

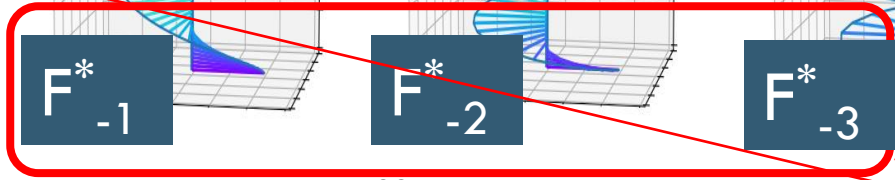
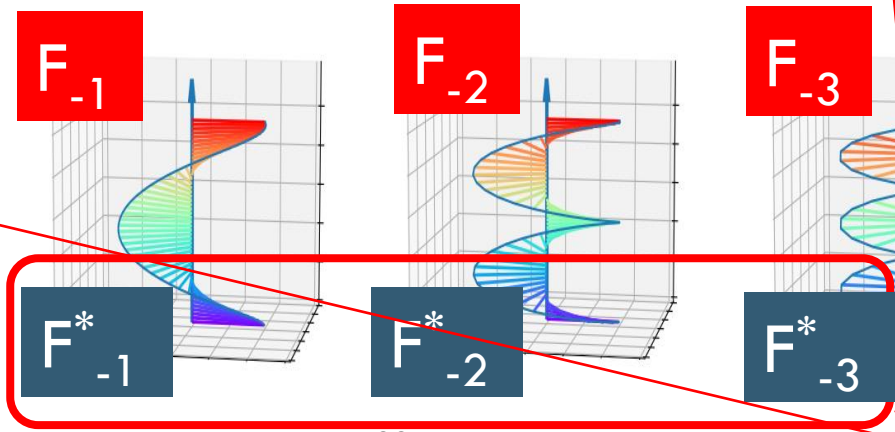
$M_+$ の  
フーリエ成分



右ねじ



$M_-$ の  
フーリエ成分



左ねじ

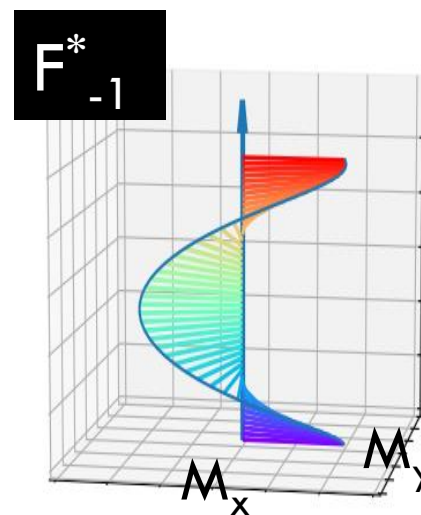
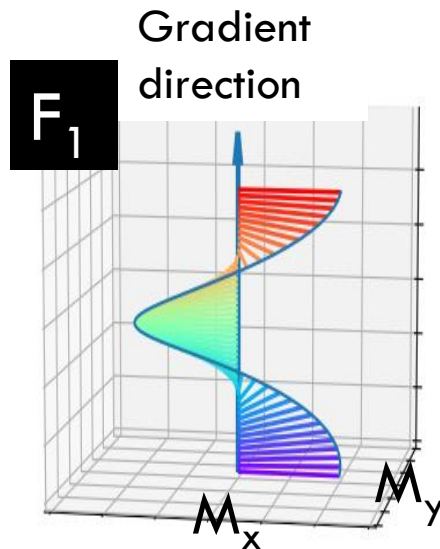
基底の取り方  
(Weigelに従う)

$$\begin{cases} M_+(\ell) = M_x + iM_y = \sum_{k=-\infty}^{\infty} F_k e^{ik\ell} \\ M_-(\ell) = M_x - iM_y = \left( \sum_{k=-\infty}^{\infty} F_k e^{ik\ell} \right)^* = \sum_{k=-\infty}^{\infty} F_{-k}^* e^{ik\ell} \end{cases}$$

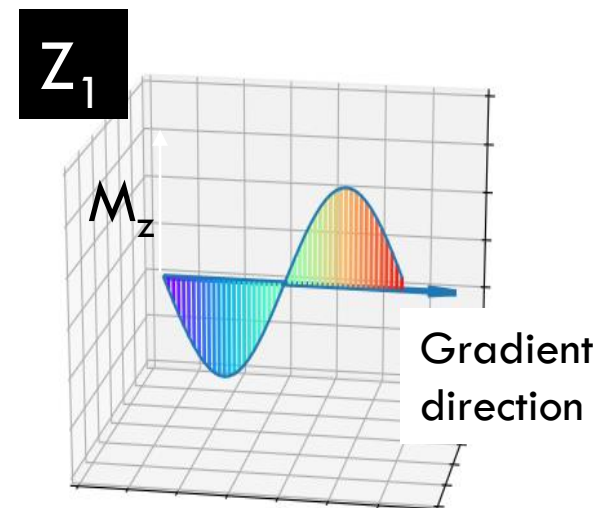
# 横磁化と縦磁化

$$\left\{ \begin{array}{l}
 M_+(\ell) = M_x + iM_y = \sum_{k=-\infty}^{\infty} F_k e^{ik\ell} \\
 M_-(\ell) = M_x - iM_y = \sum_{k=-\infty}^{\infty} F_{-k}^* e^{ik\ell} \\
 M_z(\ell) = \sum_{k=-\infty}^{\infty} Z_k e^{ik\ell} = Z_0 + 2\text{Re} \sum_{k=1}^{\infty} Z_k e^{ik\ell}
 \end{array} \right.$$

横磁化  
縦磁化



横磁化



縦磁化

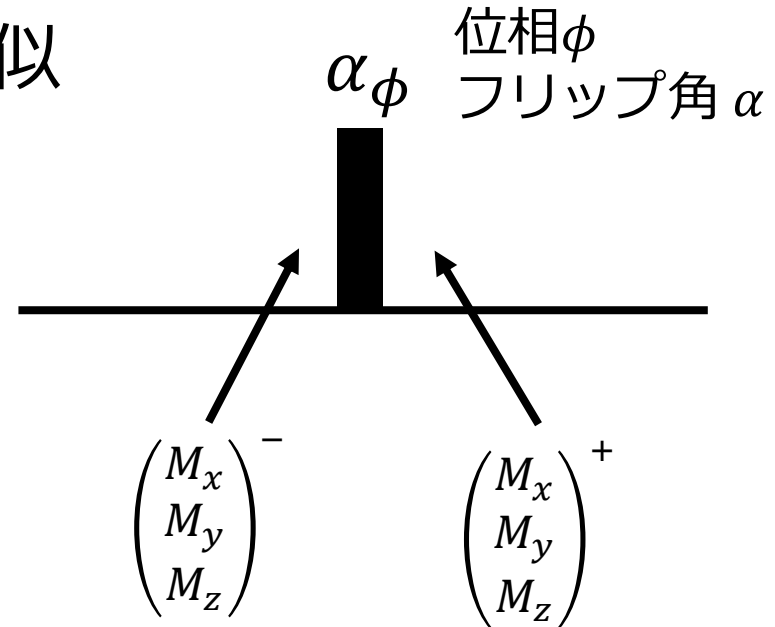
# これまでのまとめ

- EPGでは、ボクセル内の多数スピン (spin isochromats) を対象とする
- dephasing、rephasingの程度を「 $k$ =らせんのねじれ具合」で表す
- 核磁化分布 = 多数の螺旋の重ね合わせ  
→ フーリエ変換
- $k = \frac{2\pi \times \text{整数}}{\text{voxel size}}$  とすると便利である

# RF PULSEの働き:古典的位相グラフ

RFパルス≡ハードパルス近似

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}^+ = R_\phi(\alpha) \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}^-$$



$$R_\phi(\alpha) = R_z(\phi)R_x(\alpha)R_{-z}(\phi)$$

$$R_z(\phi) = \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

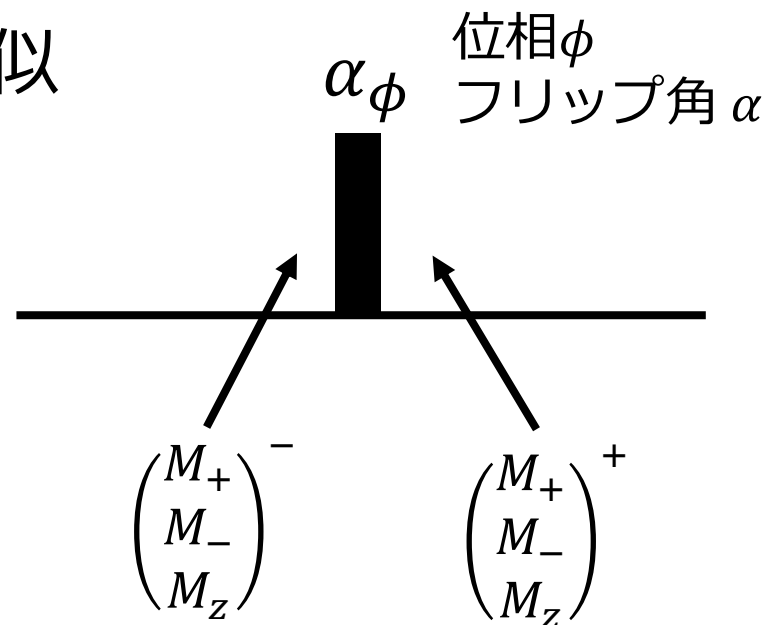
$$R_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{pmatrix}$$

核磁化のRFパルス応答は  
**行列**で表現される

# RF PULSEの働き:古典的位相グラフ

RFパルス≡ハードパルス近似

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}^+ = R_\phi(\alpha) \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}^-$$



基底変換

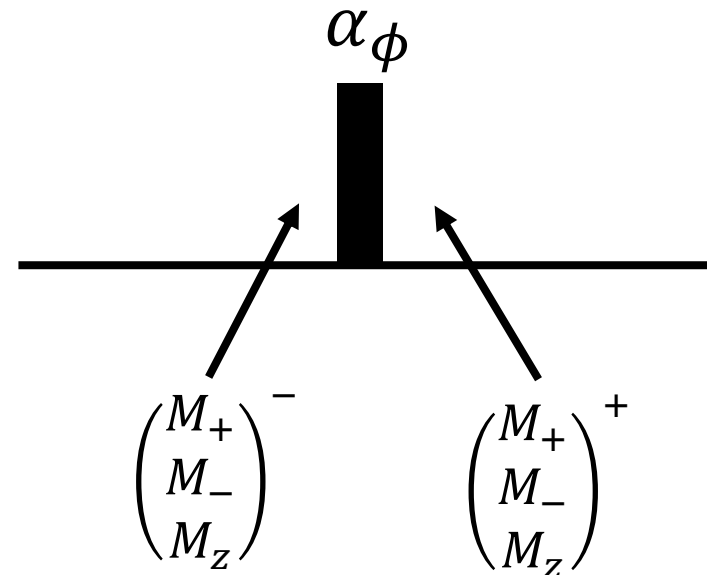
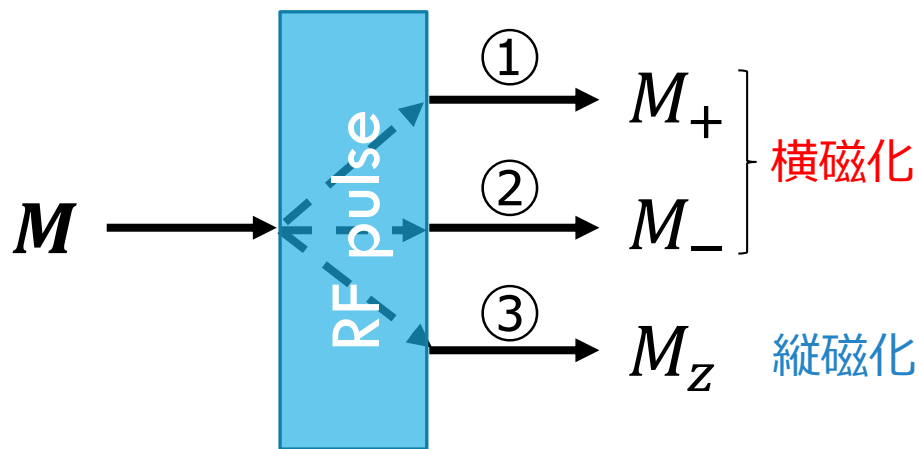
$$\begin{pmatrix} M_+ \\ M_- \\ M_z \end{pmatrix} = \begin{pmatrix} 1 & i & 0 \\ 1 & -i & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

複素表現

$$\begin{pmatrix} M_+ \\ M_- \\ M_z \end{pmatrix}^+ = \begin{pmatrix} \cos^2 \frac{\alpha}{2} & e^{i\phi} \sin^2 \frac{\alpha}{2} & -ie^{i\phi} \sin \alpha \\ e^{-i\phi} \sin^2 \frac{\alpha}{2} & \cos^2 \frac{\alpha}{2} & ie^{i\phi} \sin \alpha \\ -\frac{i}{2} e^{i\phi} \sin \alpha & \frac{i}{2} e^{i\phi} \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} M_+ \\ M_- \\ M_z \end{pmatrix}^- \equiv T \begin{pmatrix} M_+ \\ M_- \\ M_z \end{pmatrix}^-$$

# RF PULSEの働き:古典的位相グラフ

RF pulseには三つの働きがある



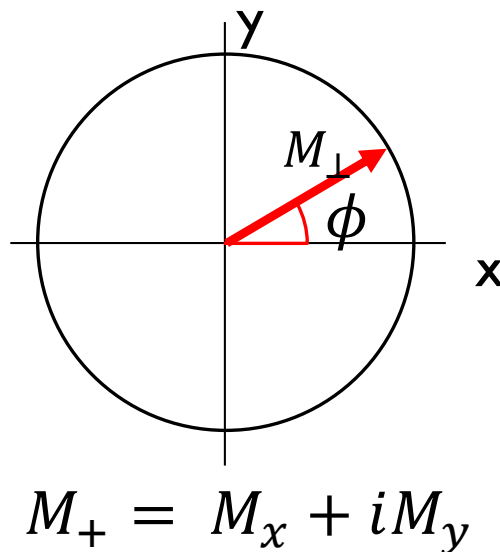
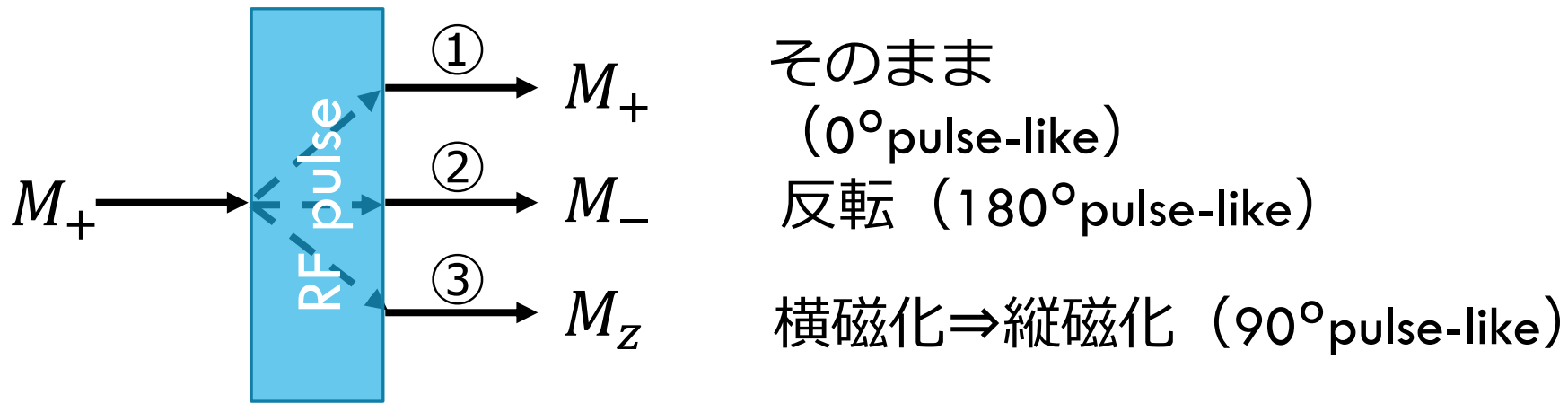
この数式から  
言えること



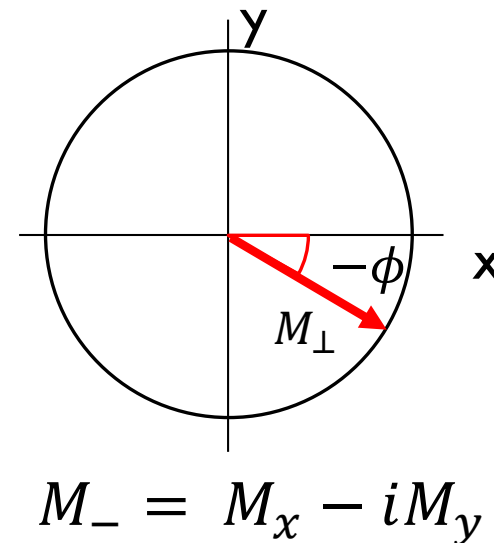
$$\begin{pmatrix} M_+ \\ M_- \\ M_z \end{pmatrix}^+ = \begin{pmatrix} \cos^2 \frac{\alpha}{2} & e^{i\phi} \sin^2 \frac{\alpha}{2} & -ie^{i\phi} \sin \alpha \\ e^{-i\phi} \sin^2 \frac{\alpha}{2} & \cos^2 \frac{\alpha}{2} & ie^{i\phi} \sin \alpha \\ -\frac{i}{2} e^{i\phi} \sin \alpha & \frac{i}{2} e^{i\phi} \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} M_+ \\ M_- \\ M_z \end{pmatrix}^- \equiv T \begin{pmatrix} M_+ \\ M_- \\ M_z \end{pmatrix}^-$$

# RF PULSEの働き:古典的位相グラフ

RF pulseには三つの働きがある



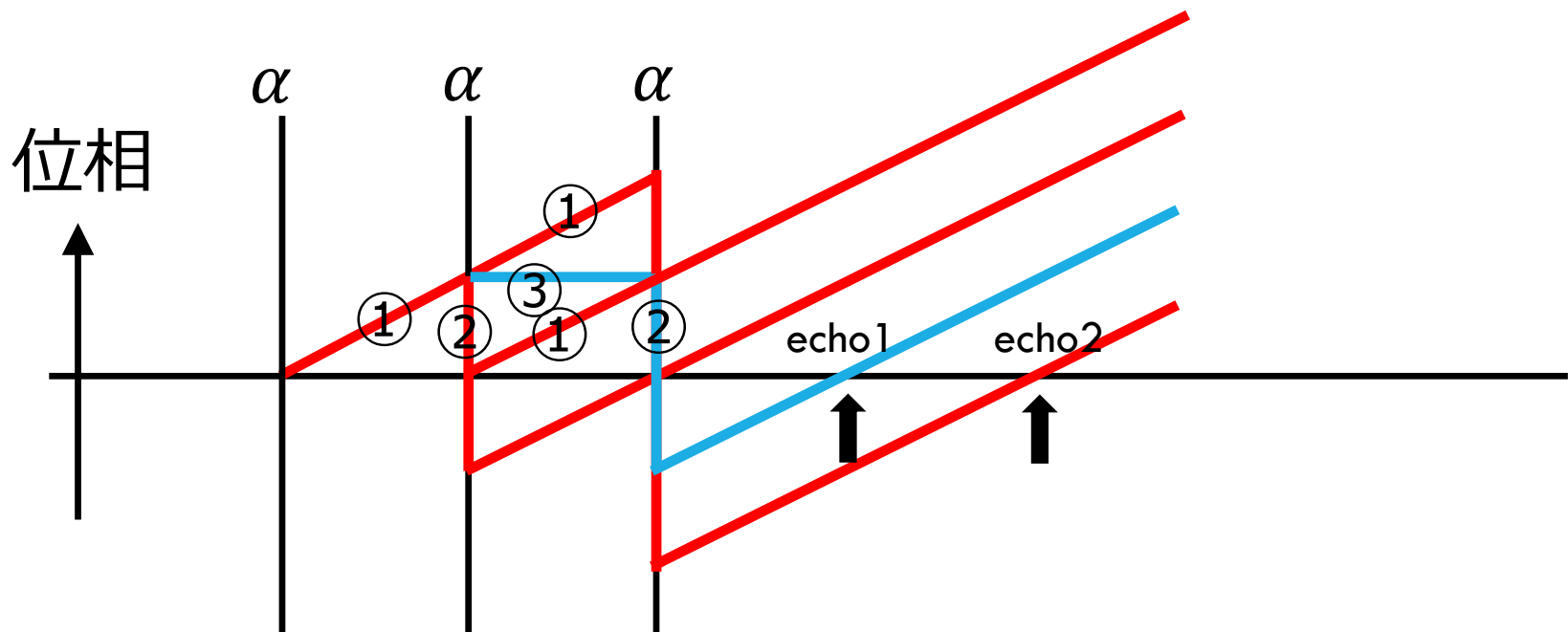
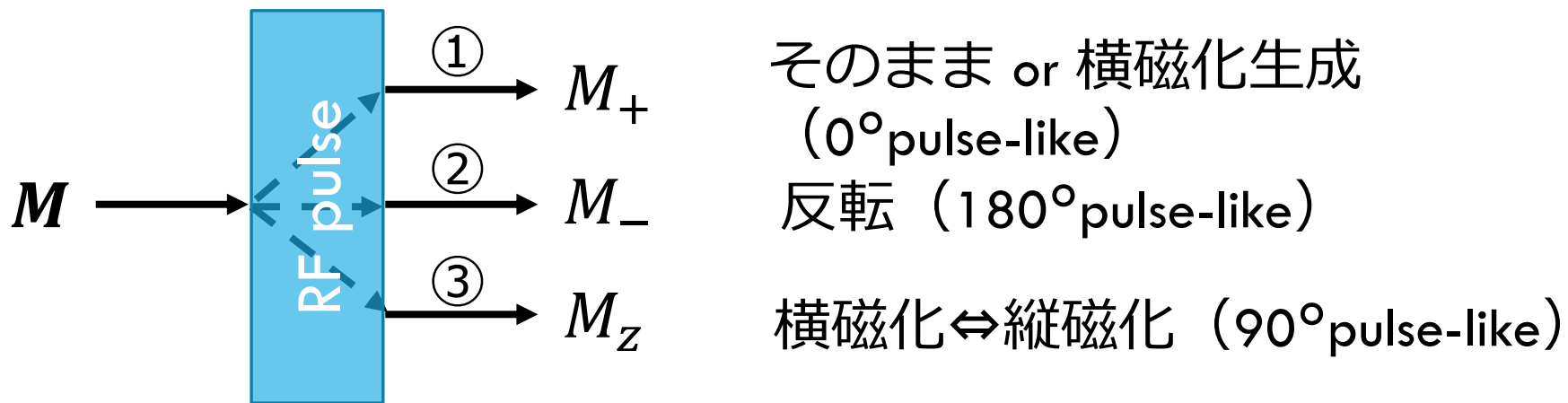
②反転  
 $180^\circ$ -like



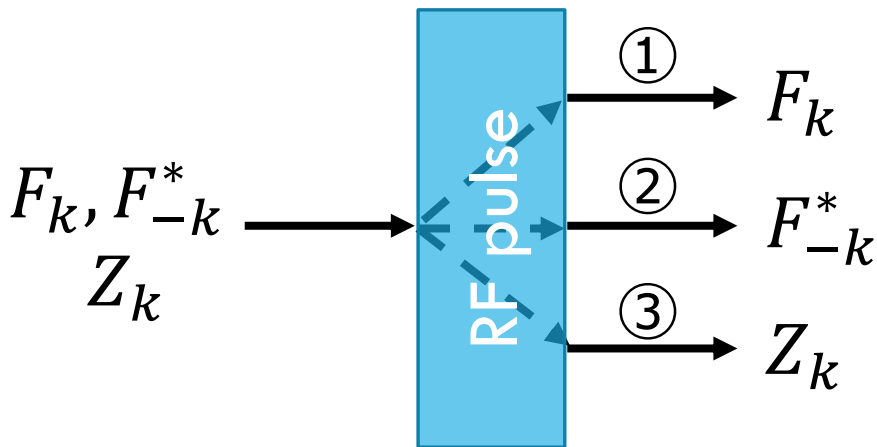


# RF PULSEの働き:古典的位相グラフ

RF pulseには三つの働きがある



# RF PULSEの働き: 拡張位相グラフ



そのまま or 横磁化生成  
(0° pulse-like)

反転 (180° pulse-like)

横磁化 ⇔ 縦磁化 (90° pulse-like)

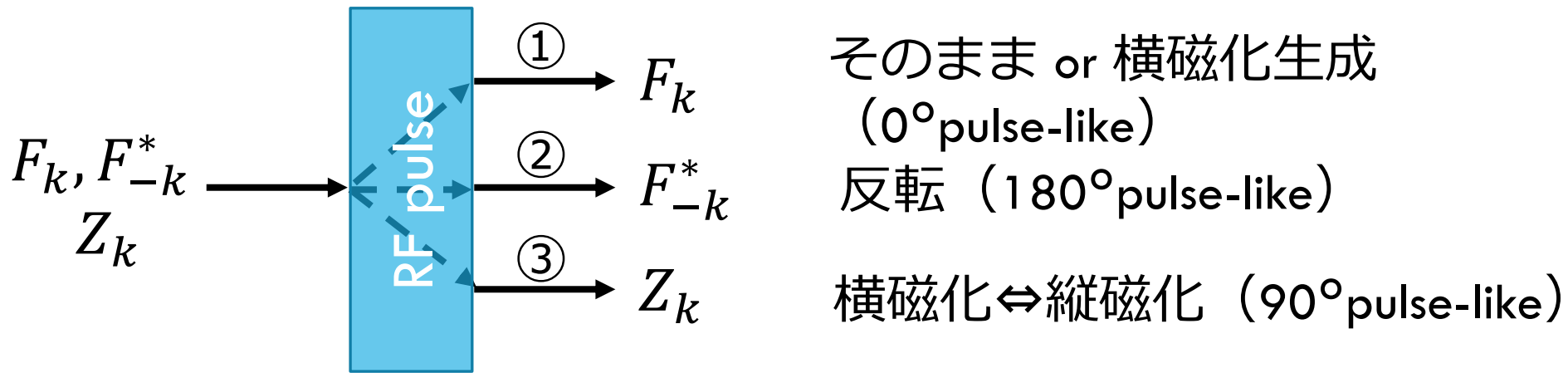
$$\begin{cases} M_+(\ell) = \sum_{k=-\infty}^{\infty} F_k e^{ik\ell} \\ M_-(\ell) = \sum_{k=-\infty}^{\infty} F_{-k}^* e^{ik\ell} \\ M_z(\ell) = \sum_{k=-\infty}^{\infty} Z_k e^{ik\ell} \end{cases}$$

フーリエ成分に関しても  
全く同じ関係が成り立つ

$$\begin{cases} \begin{pmatrix} M_+ \\ M_- \\ M_z \end{pmatrix}^+ = T \begin{pmatrix} M_+ \\ M_- \\ M_z \end{pmatrix}^- & \text{実空間} \\ \begin{pmatrix} F_k \\ F_{-k}^* \\ Z_k \end{pmatrix}^+ = T \begin{pmatrix} F_k \\ F_{-k}^* \\ Z_k \end{pmatrix}^- & \text{フーリエ空間} \end{cases}$$

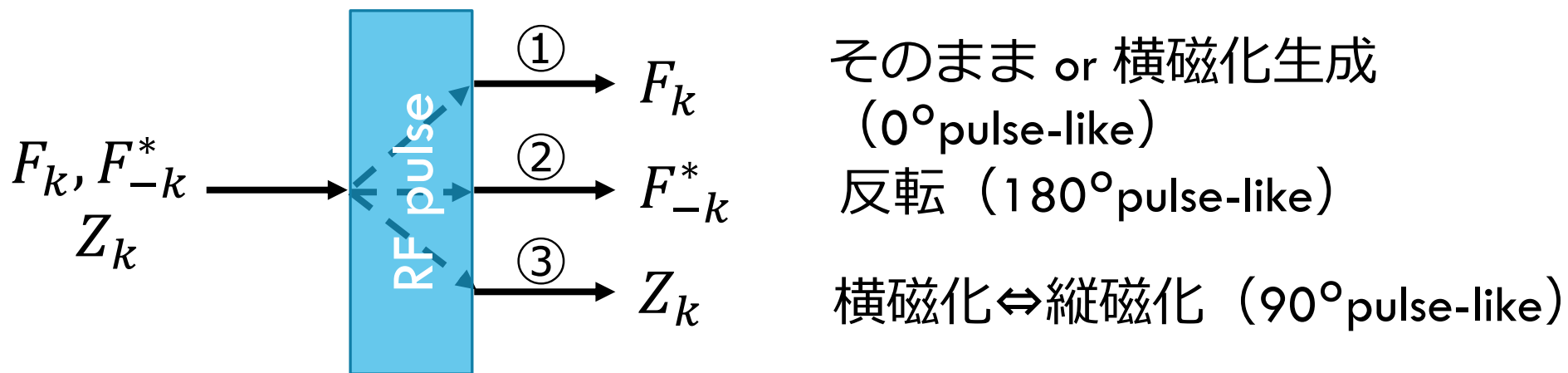
同じ回転行列  $T$  で表現できる  
※基底の取り方によってはそう  
ならない

# RF PULSEの働き: 拡張位相グラフ



数学的に三つに分かれるのは分かるが...  
 どうもしっくりこない  
 特に「**反転**」作用について

# RF PULSEの働き: 拡張位相グラフ

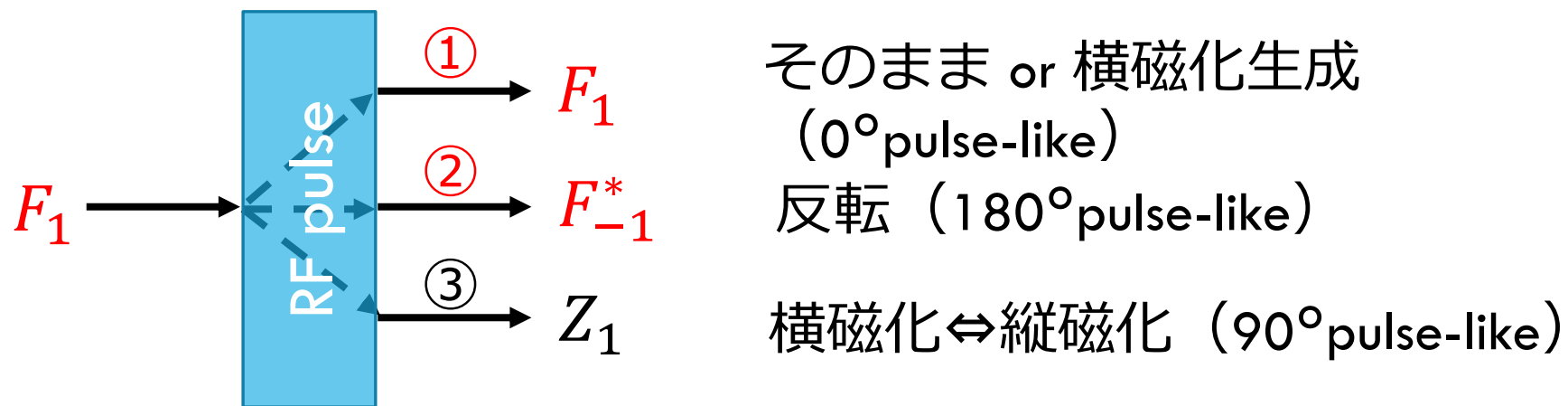


例えば  $\phi = 0, \alpha = 90^\circ$  の場合

$$\begin{pmatrix} F_k \\ F_{-k}^* \\ Z_k \end{pmatrix}^+ = \begin{pmatrix} 0.5 & -0.5 & 1 \\ -0.5 & 0.5 & 1 \\ -0.5 & -0.5 & 0 \end{pmatrix} \begin{pmatrix} F_k \\ F_{-k}^* \\ Z_k \end{pmatrix}^-$$

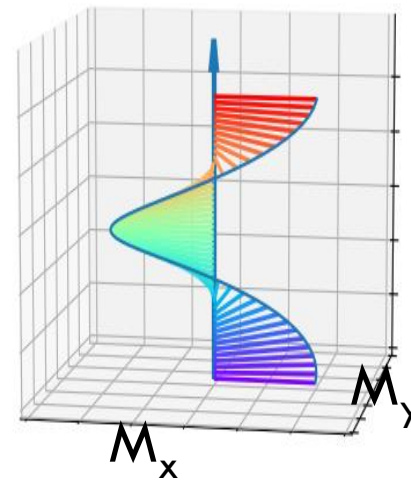
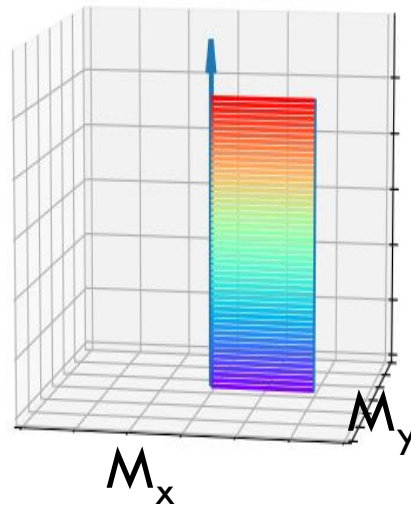
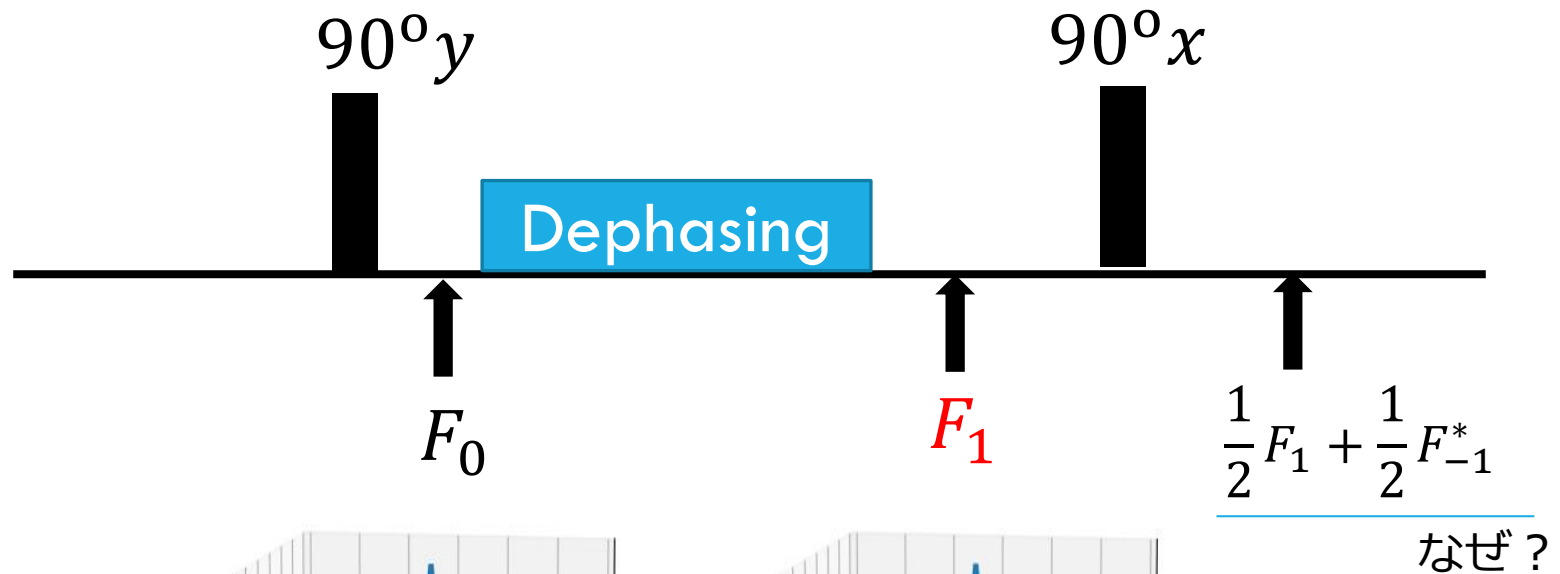
90°パルスでさえ反転作用がある

# RF PULSEの働き: 拡張位相グラフ



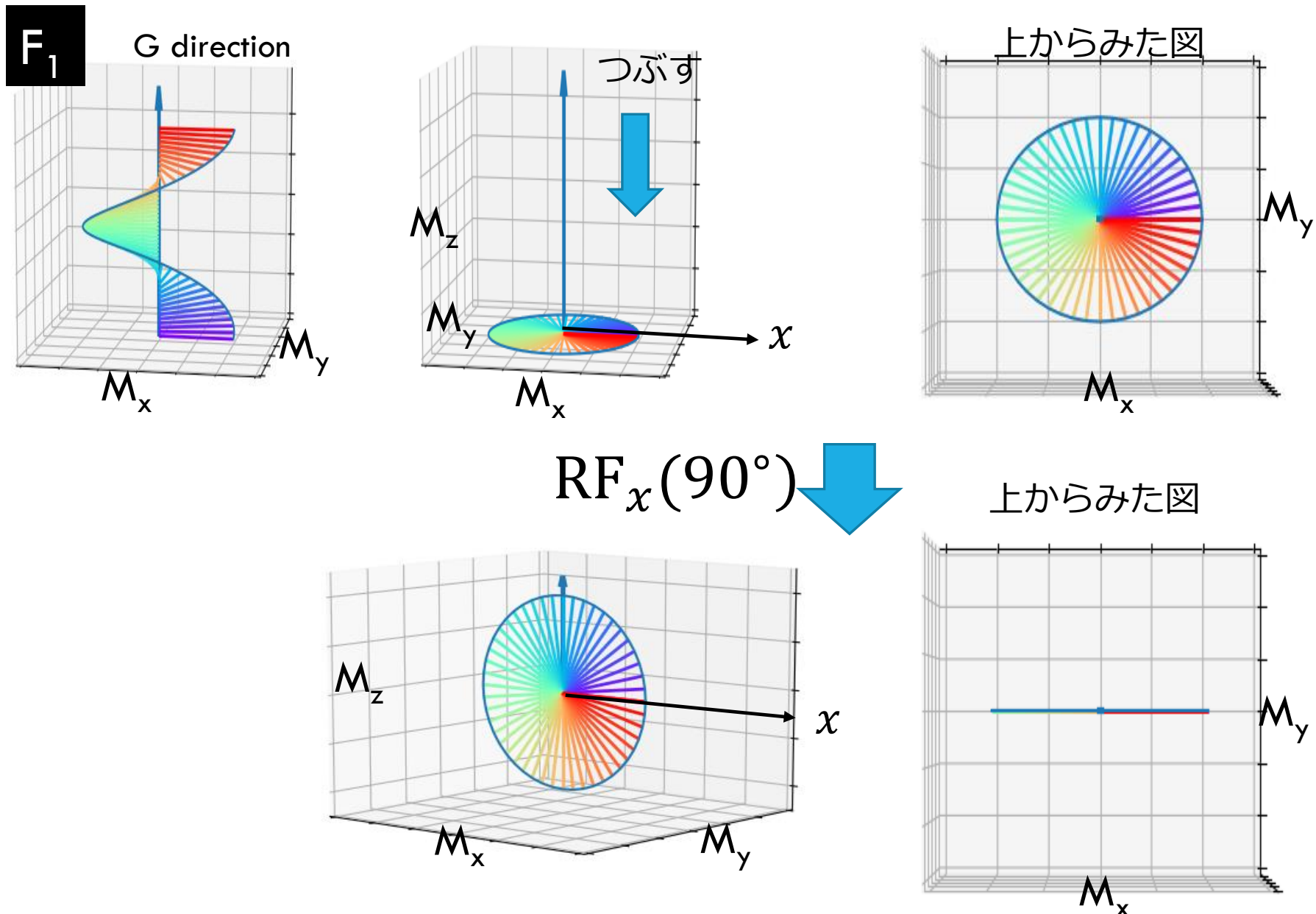
単一のスピンでは理解できないが、  
多数のスピンを考えると理解できる

# RF PULSEの働き：反転

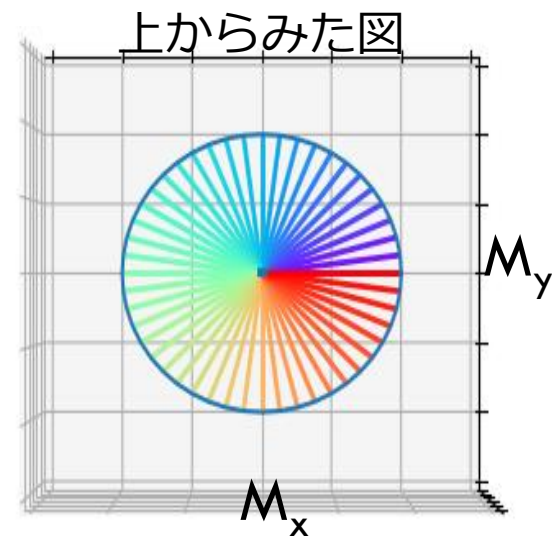
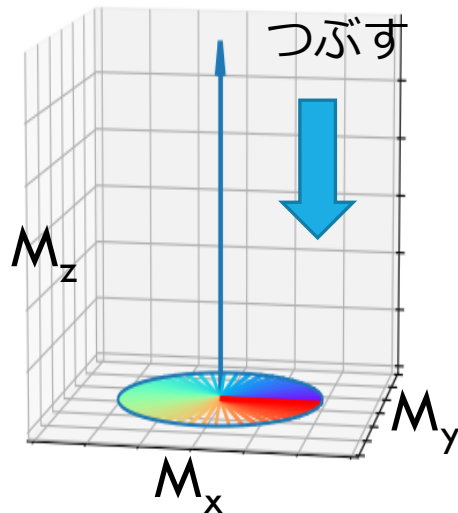
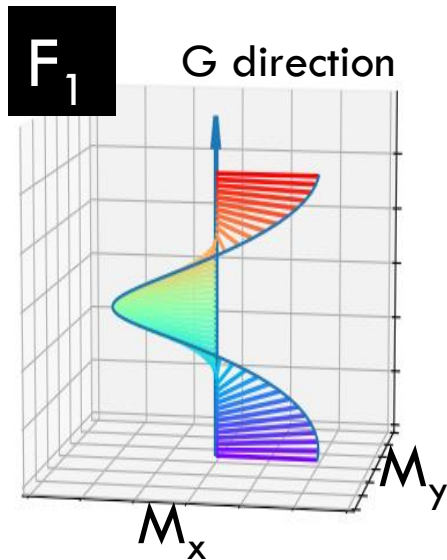


ここを初期状態  
と考える

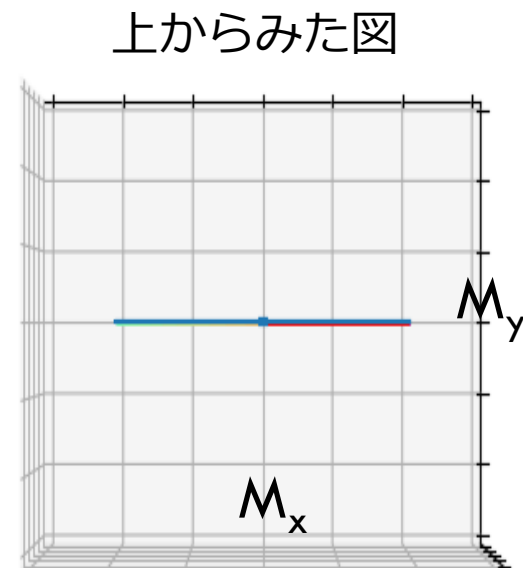
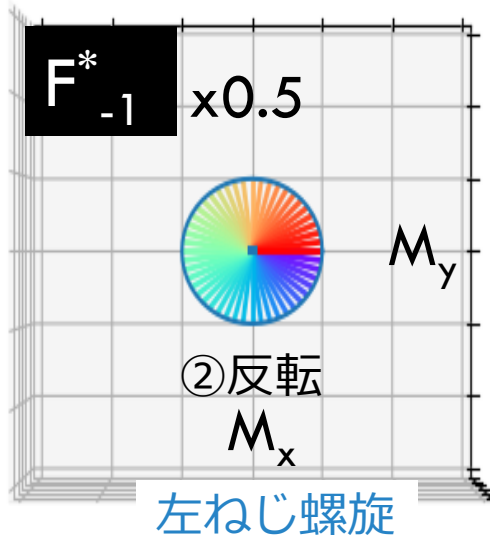
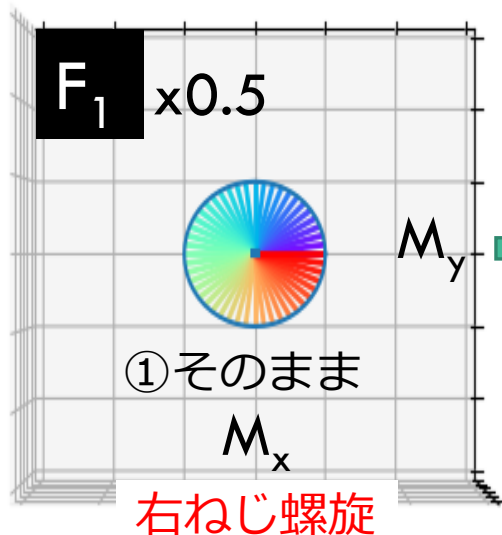
# RF PULSEの働き：反転



# RF PULSEの働き：反転

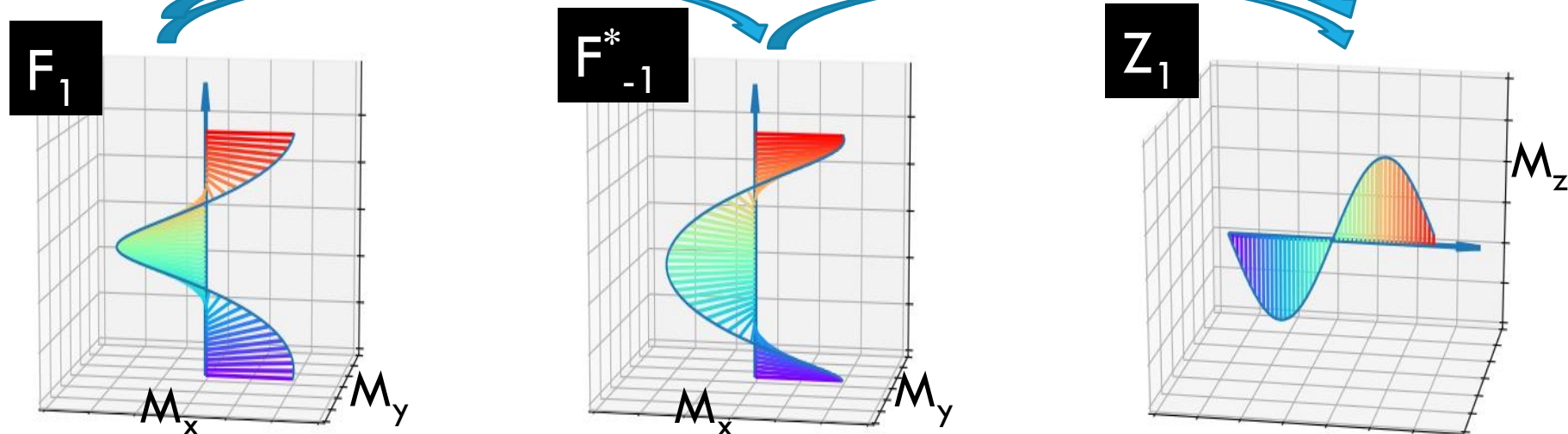
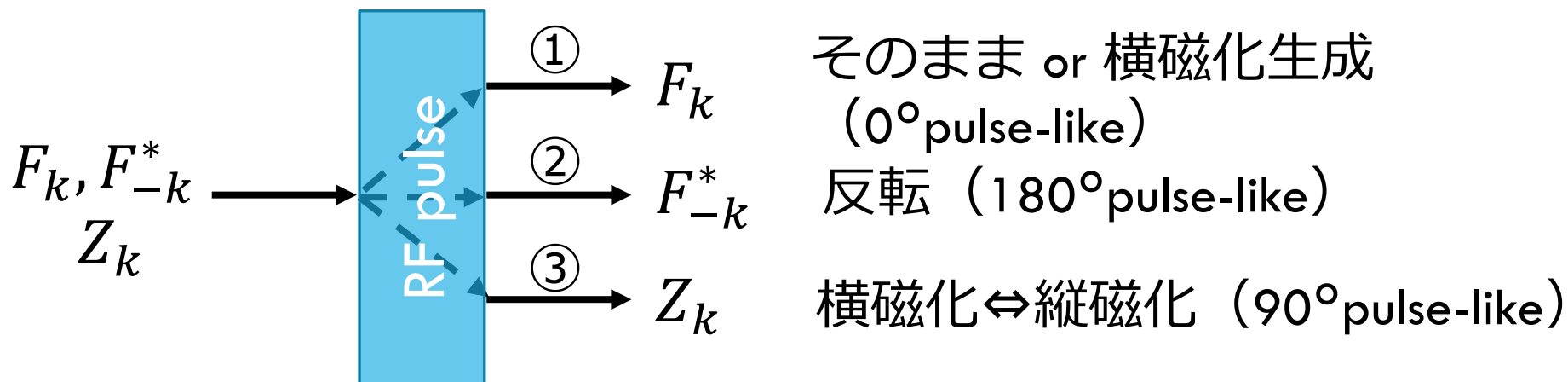


RF<sub>x</sub>(90°)





# RF PULSEの働き: 拡張位相グラフ



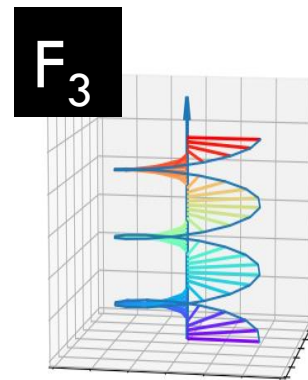
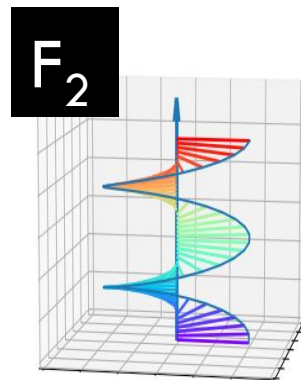
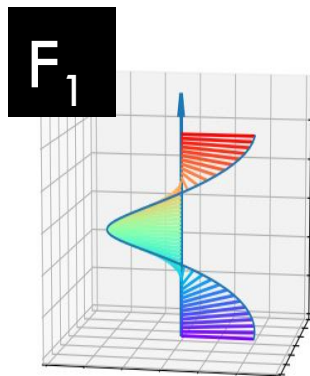
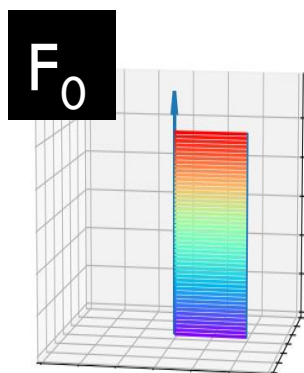
RFパルス=同じ $|k|$ どうしのフーリエ成分を混合する

フーリエ基底="state"

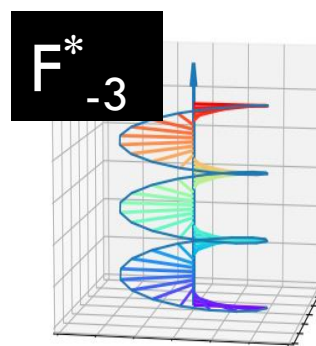
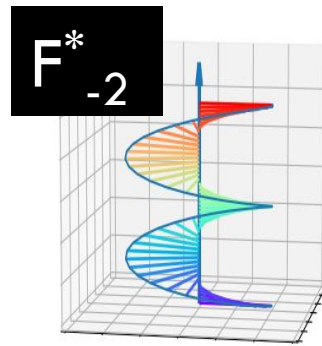
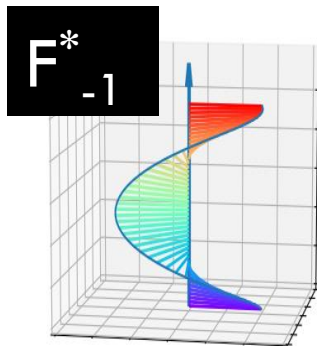
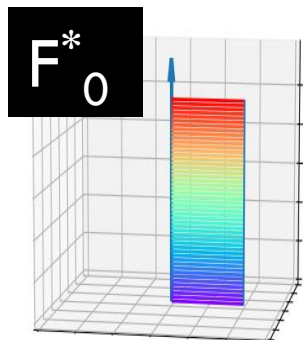
# EPG "STATES"

"configuration states"

横磁化 ( $M_+$ ,  $M_-$ )

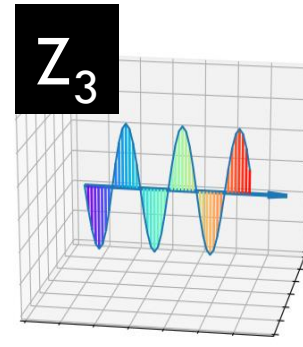
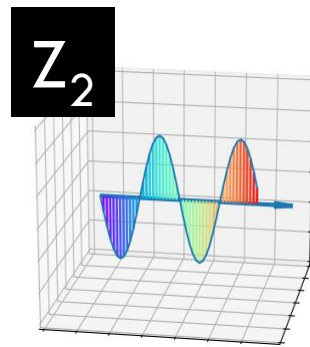
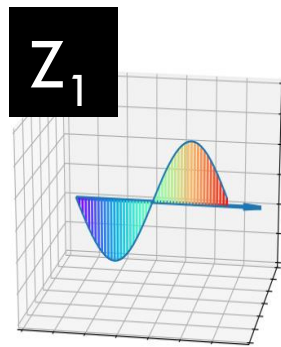
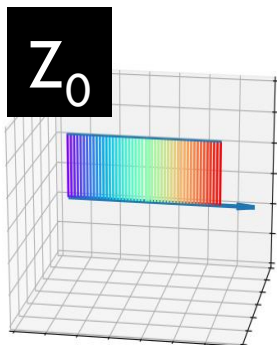


... 右ねじ螺旋



... 左ねじ螺旋

縦磁化 ( $M_z$ )



...

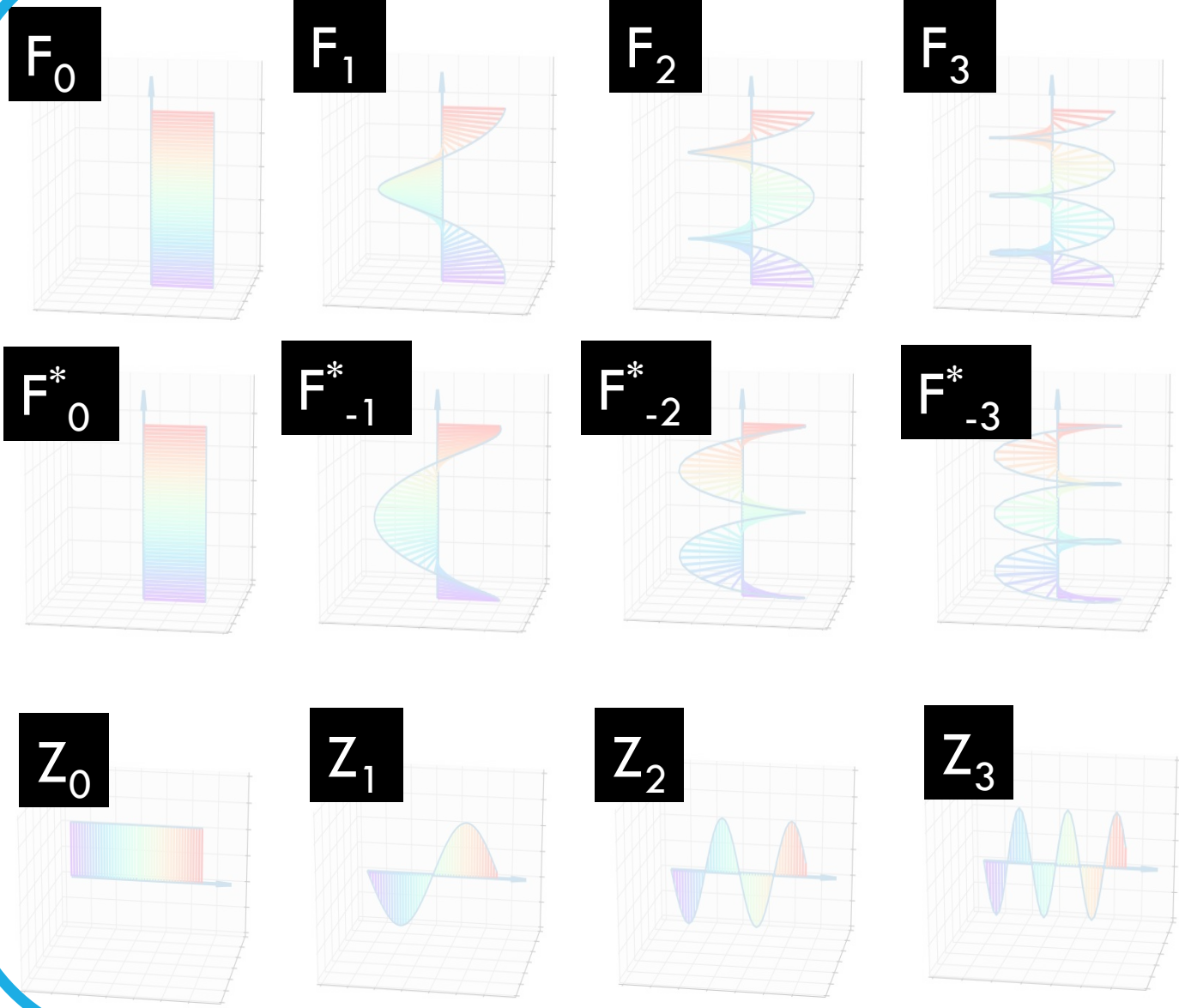
フーリエ基底="state"

# EPG "STATES"

"configuration states"

横磁化 ( $M_+, M_-$ )

縦磁化 ( $M_z$ )



$= \Omega$

... 右ねじ螺旋

... 左ねじ螺旋

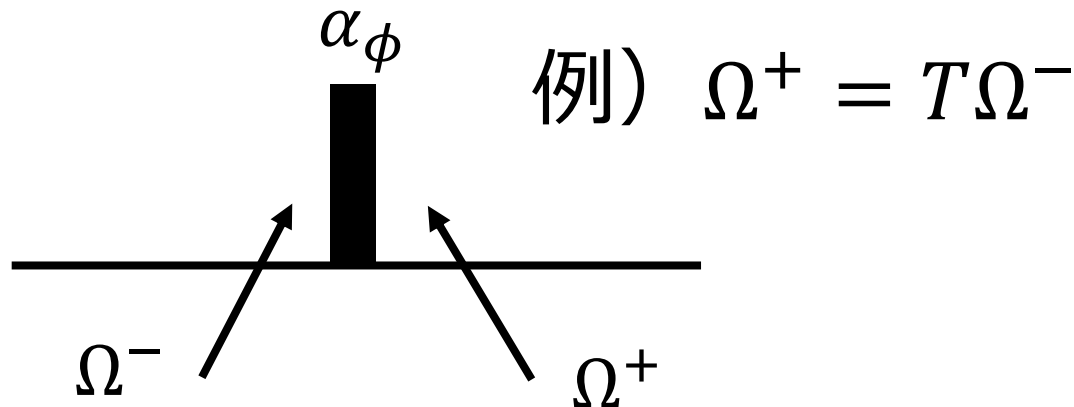
...

## EPG “STATES”

- 行列で管理

$$\Omega = \begin{pmatrix} F_0 & F_1 & F_2 & F_3 & \cdots \\ F_0^* & F_{-1}^* & F_{-2}^* & F_{-3}^* & \cdots \\ Z_0 & Z_1 & Z_2 & Z_3 & \cdots \end{pmatrix} \begin{matrix} M_+ \\ M_- \\ M_z \end{matrix} \begin{matrix} \text{右ねじ螺旋} \\ \text{左ねじ螺旋} \\ \text{縦磁化} \end{matrix}$$

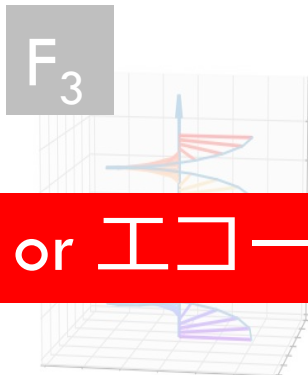
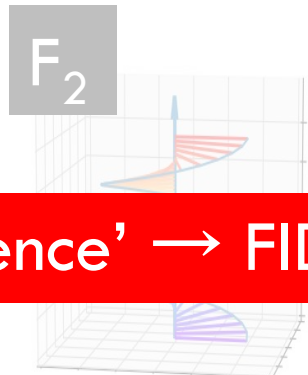
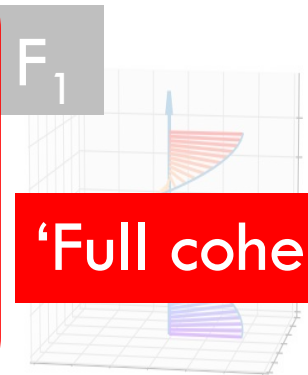
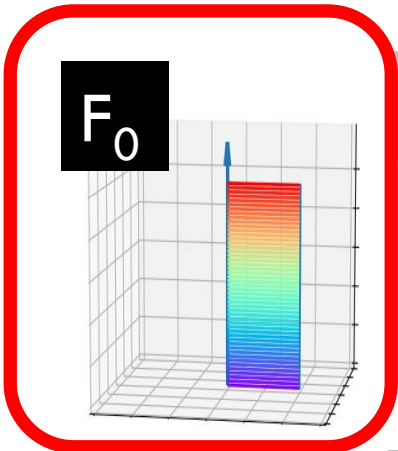
- RF pulse や Gradient、緩和等の効果も行列で表現



# EPG "STATES"

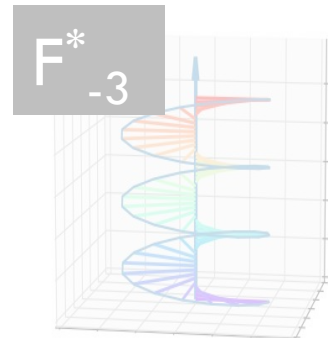
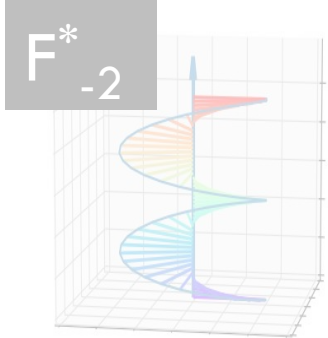
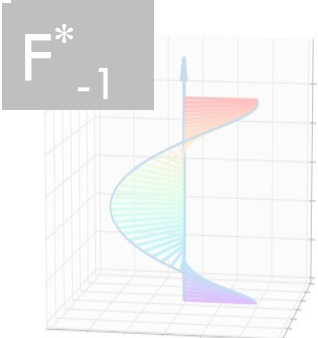
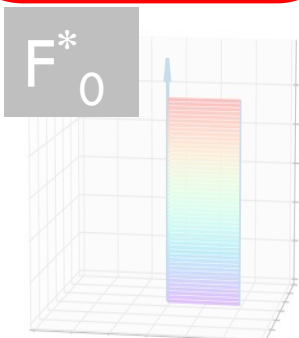
横磁化 ( $M_+$ ,  $M_-$ )

縦磁化 ( $M_z$ )

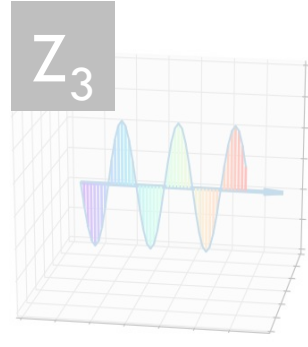
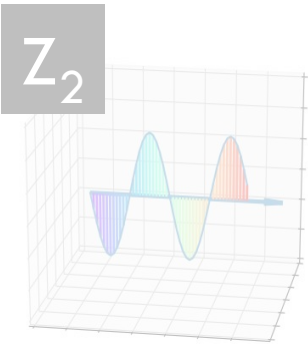
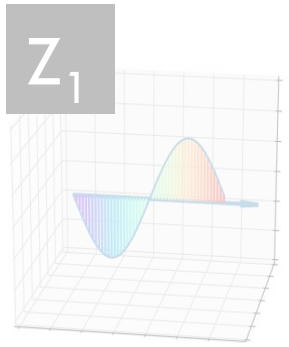
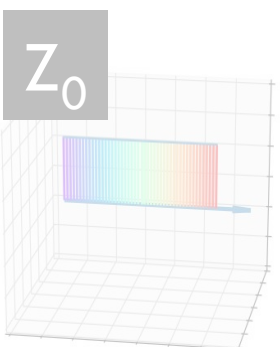


'Full coherence' → FID or  $\Gamma \square -$

... 右ねじ螺旋



... 左ねじ螺旋

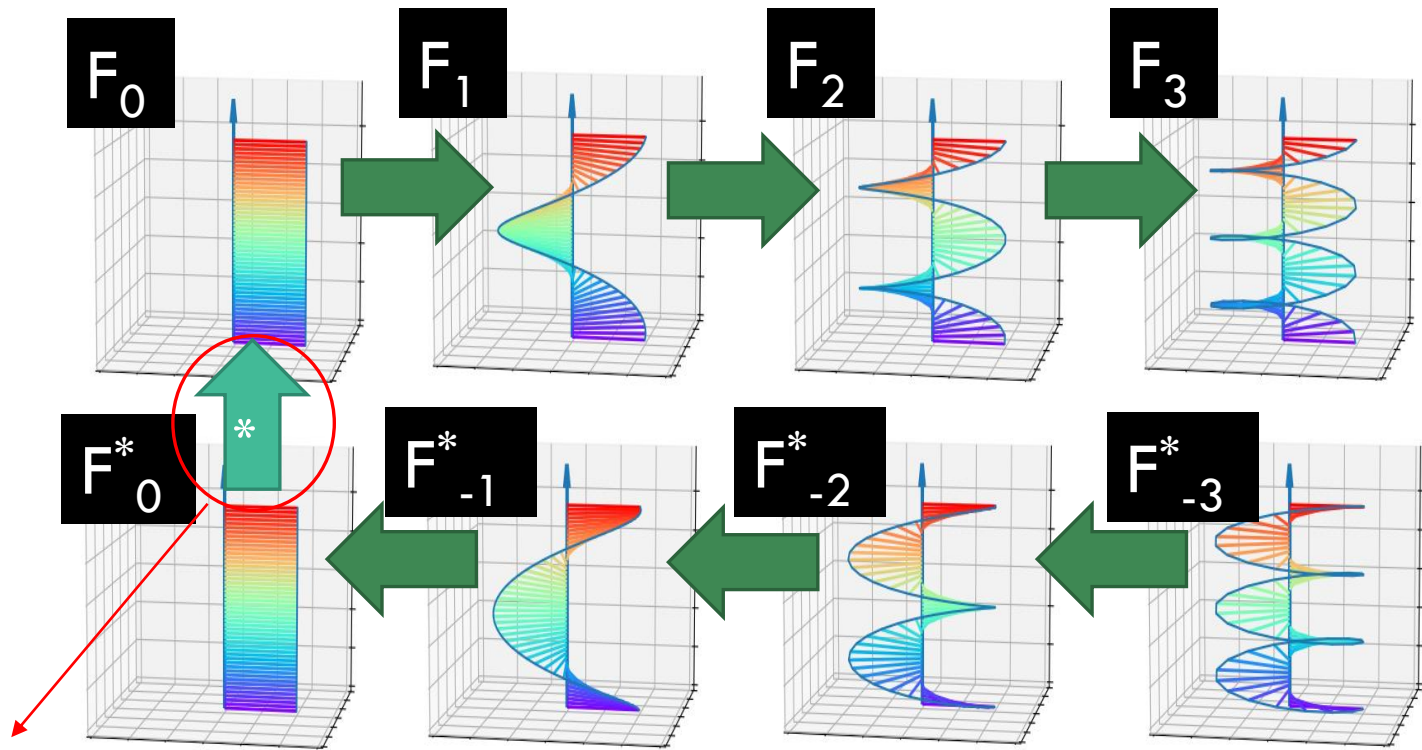


...

dephasing

EPG "STATES"

横磁化 ( $M_+$ ,  $M_-$ )



... 右ねじ螺旋

... 左ねじ螺旋

“複素共役”をとることを忘れずに

“S”演算子

$$S(+1): F_k \rightarrow F_{k+1}$$

$$Z_k \rightarrow Z_k$$

dephasing

## EPG “STATES”

$$S(+1): F_k \rightarrow F_{k+1}$$

$$Z_k \rightarrow Z_k$$

“複素共役”

$$\Omega^- = \begin{pmatrix} F_0 \rightarrow F_1 \rightarrow F_2 \rightarrow F_3 & \cdots \\ * F_0^* \leftarrow F_{-1}^* \leftarrow F_{-2}^* \leftarrow F_{-3}^* & \cdots \\ Z_0 & Z_1 & Z_2 & Z_3 & \cdots \end{pmatrix} \begin{matrix} M_+ \\ M_- \\ M_z \end{matrix}$$

右ねじ螺旋  
左ねじ螺旋  
縦磁化

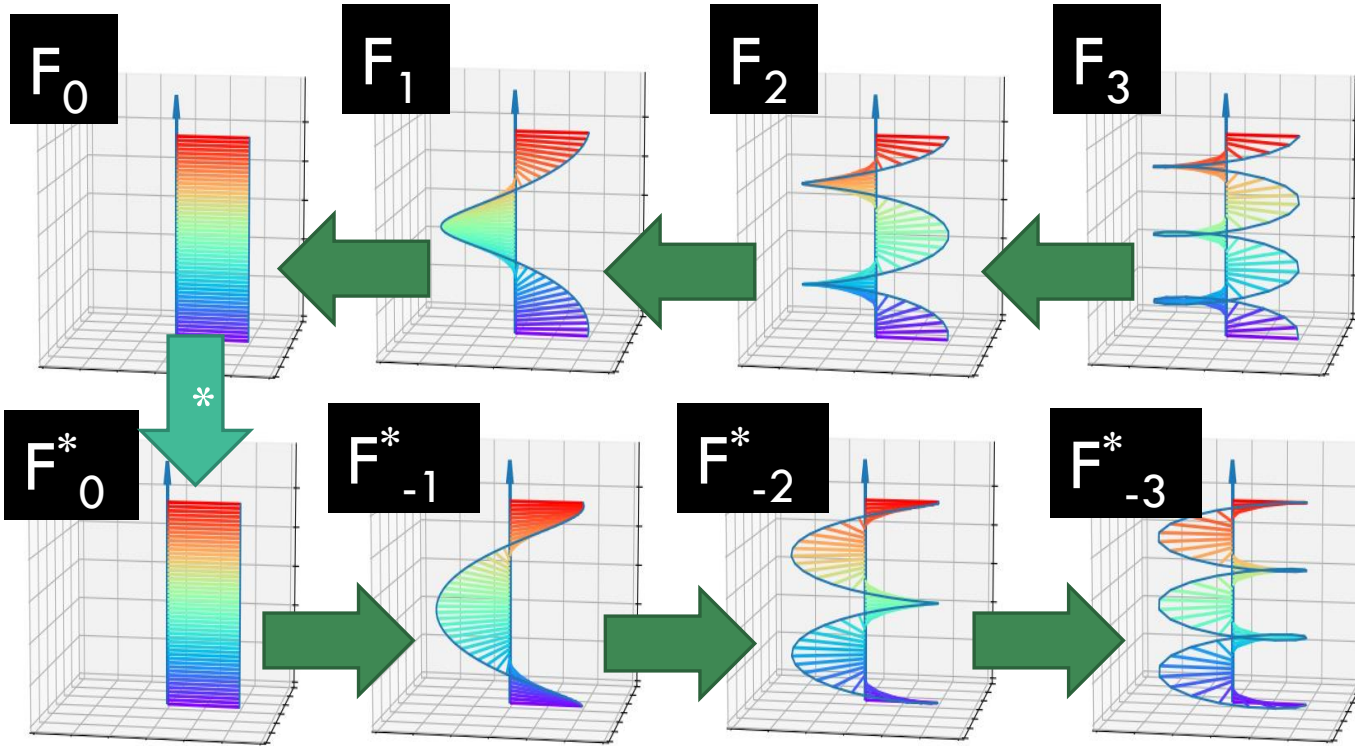
$$\Omega^+ = S(+1)\Omega^-$$

$$= \begin{pmatrix} F_{-1} & F_0 & F_1 & F_2 & \cdots \\ F_{-1}^* & F_{-2}^* & F_{-3}^* & F_{-4}^* & \cdots \\ Z_0 & Z_1 & Z_2 & Z_3 & \cdots \end{pmatrix}$$

rephasing

# EPG "STATES"

横磁化 ( $M_+, M_-$ )



... 右ねじ

... 左ねじ

“S”演算子

$$S(-1): F_k \rightarrow F_{k-1}$$

$$Z_k \rightarrow Z_k$$

...

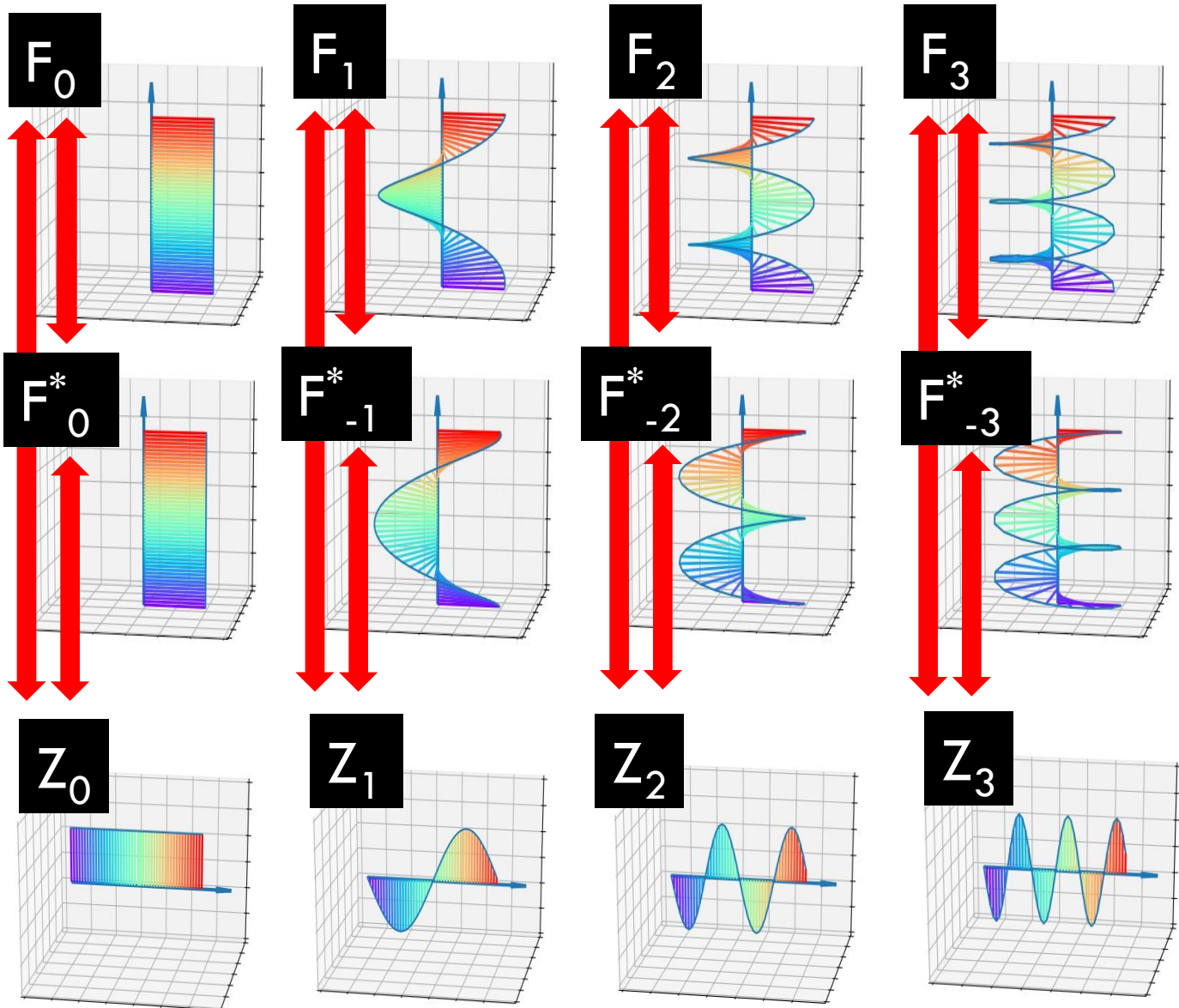


RF pulse

EPG "STATES"

横磁化 ( $M_+$ ,  $M_-$ )

縦磁化 ( $M_z$ )



... 右ねじ

... 左ねじ

...

RF pulse

## EPG “STATES”

“T”演算子

$$T = \begin{pmatrix} \cos^2 \frac{\alpha}{2} & e^{i\phi} \sin^2 \frac{\alpha}{2} & -ie^{i\phi} \sin \alpha \\ e^{-i\phi} \sin^2 \frac{\alpha}{2} & \cos^2 \frac{\alpha}{2} & ie^{i\phi} \sin \alpha \\ -\frac{i}{2} e^{i\phi} \sin \alpha & \frac{i}{2} e^{i\phi} \sin \alpha & \cos \alpha \end{pmatrix}$$

$$\Omega^+ = T\Omega^-$$

## T1, T2緩和

## EPG “STATES”

## “E”演算子

$$E(\tau, T_1, T_2) = \begin{pmatrix} E_2 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{pmatrix} \text{ for } k \neq 0$$

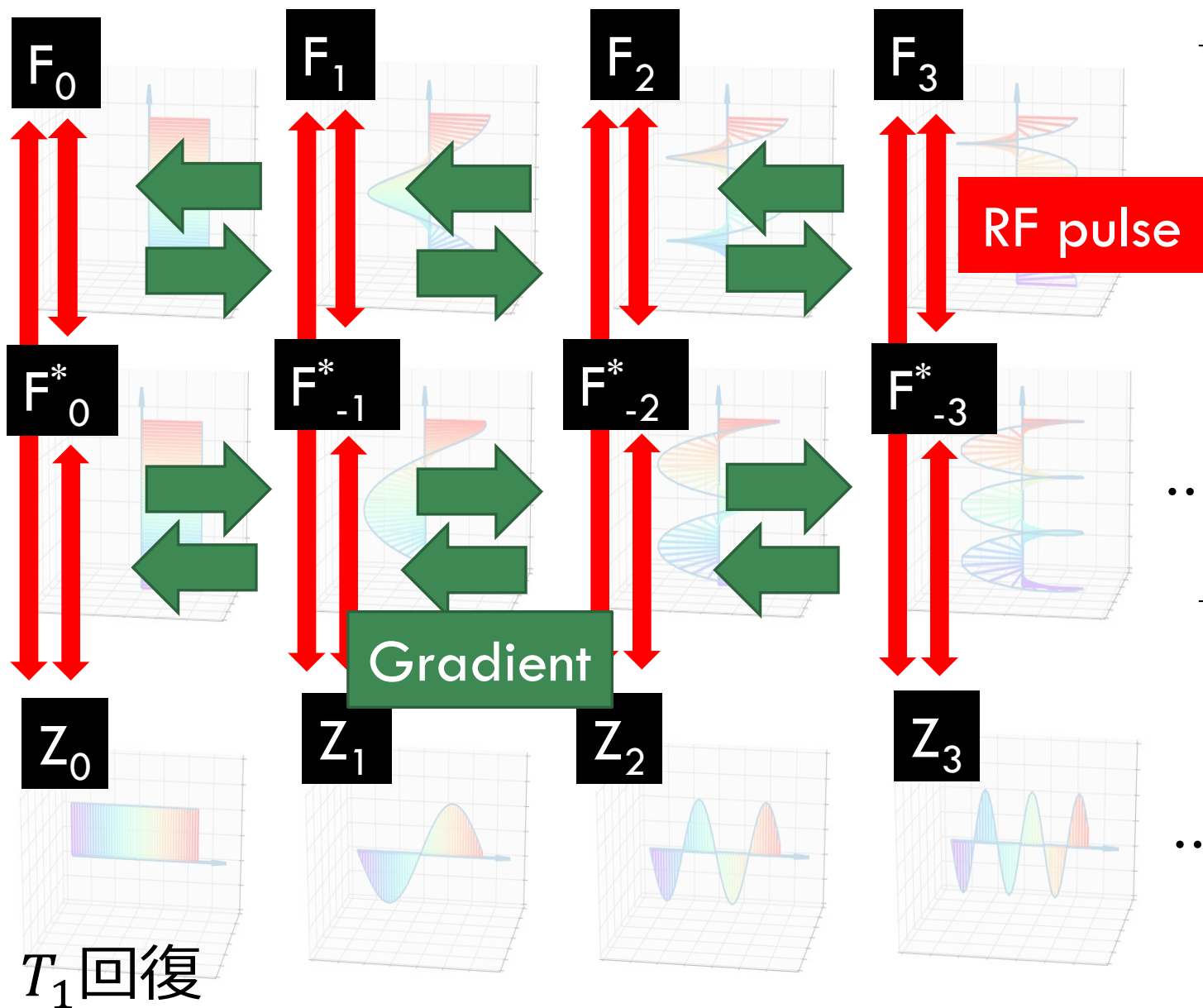
$$E_1 = \exp\left(-\frac{\tau}{T_1}\right), E_2 = \exp\left(-\frac{\tau}{T_2}\right)$$

$$\begin{pmatrix} F_0 \\ F_0^* \\ Z_0 \end{pmatrix}^+ = E \begin{pmatrix} F_0 \\ F_0^* \\ Z_0 \end{pmatrix}^- + \begin{pmatrix} 0 \\ 0 \\ M_0(1 - E_1) \end{pmatrix} \text{ for } k = 0$$

# EPG "STATES"

橫磁化 ( $M_+$ ,  $M_-$ )

縱磁化 ( $M_z$ )



$T_2$  減衰

...

$T_1$  減衰

...

$T_1$  回復

# 实例

# CPMG (理想)

$90^\circ_y$   $180^\circ_x$   $180^\circ_x$   $180^\circ_x$   $180^\circ_x$



演算子  $T$   $S$   $T$   $S$   $S$   $T$   $S$   $S$   $T$   $S$   $S$   $T$

• RFパルス

$$T_{90y} = \begin{pmatrix} 0.5 & -0.5 & 1 \\ -0.5 & 0.5 & 1 \\ -0.5 & -0.5 & 0 \end{pmatrix}$$

• シフト演算子

$$S(+1): F_k \rightarrow F_{k+1}$$

$$Z_k \rightarrow Z_k$$

$$T_{180x} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

• 緩和項は無視

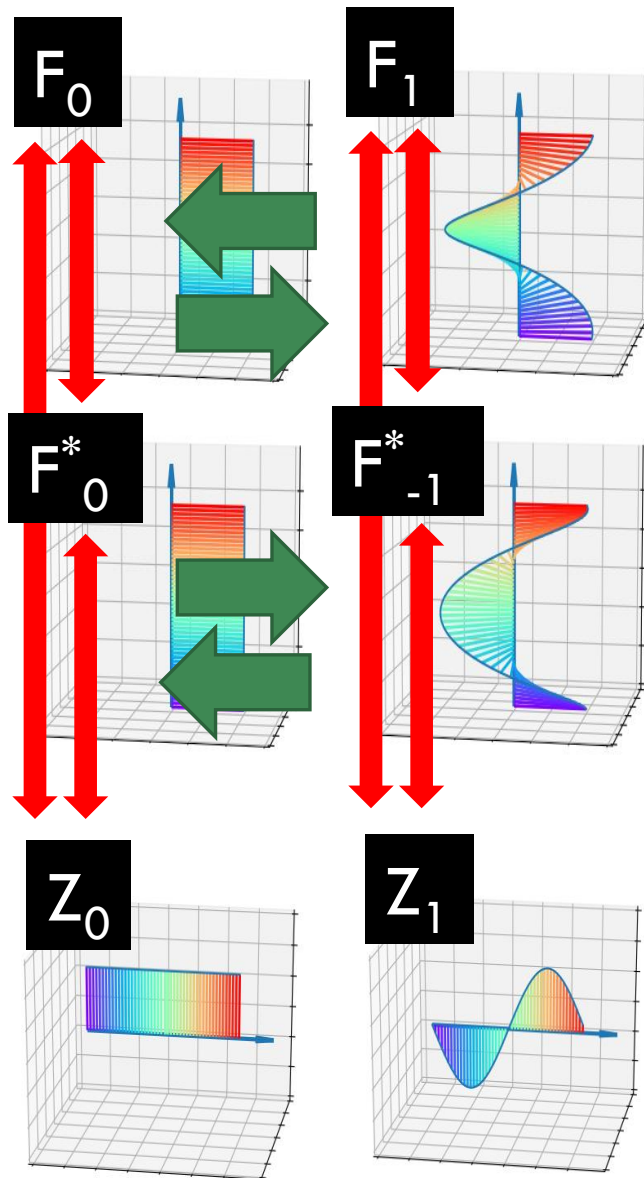


- $F_k$  と  $F_{-k}^*$  を交換
- $Z_k$  を符号反転

# CPMG (理想)

横磁化 ( $M_+$ ,  $M_-$ )

縦磁化 ( $M_z$ )

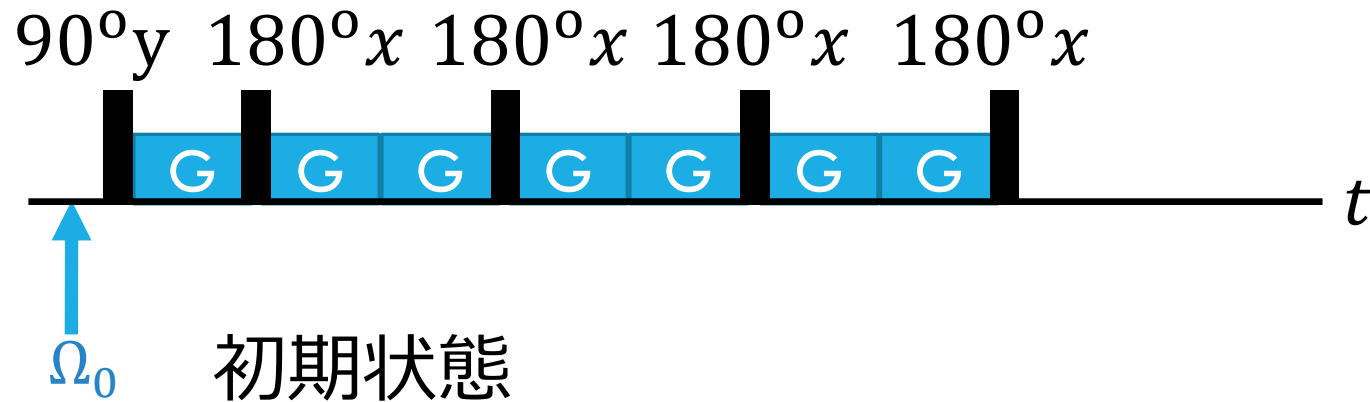


このシーケンスの場合  
 $|k| \geq 2$ の項はすべてゼロ

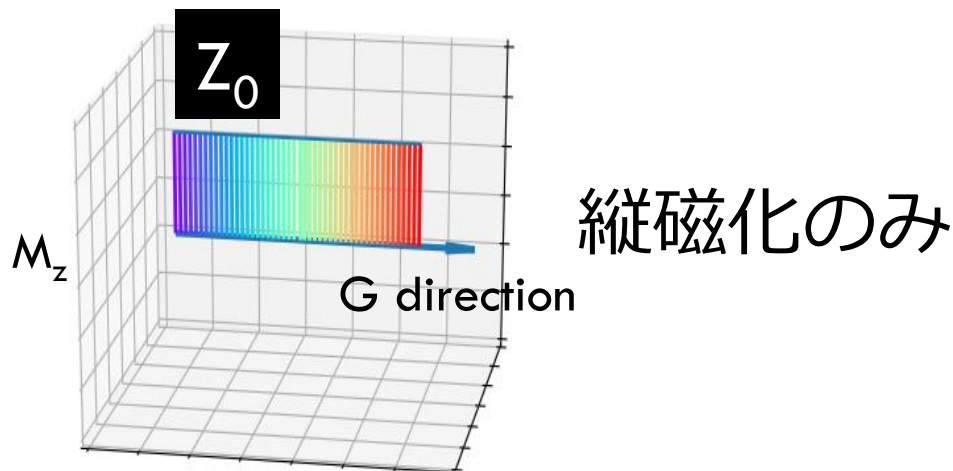
$$\Omega = \begin{pmatrix} F_0 & F_1 & 0 & \dots \\ F_0^* & F_{-1}^* & 0 & \dots \\ Z_0 & Z_1 & 0 & \dots \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} F_0 & F_1 \\ F_0^* & F_{-1}^* \\ Z_0 & Z_1 \end{pmatrix}$$

# CPMG (理想)

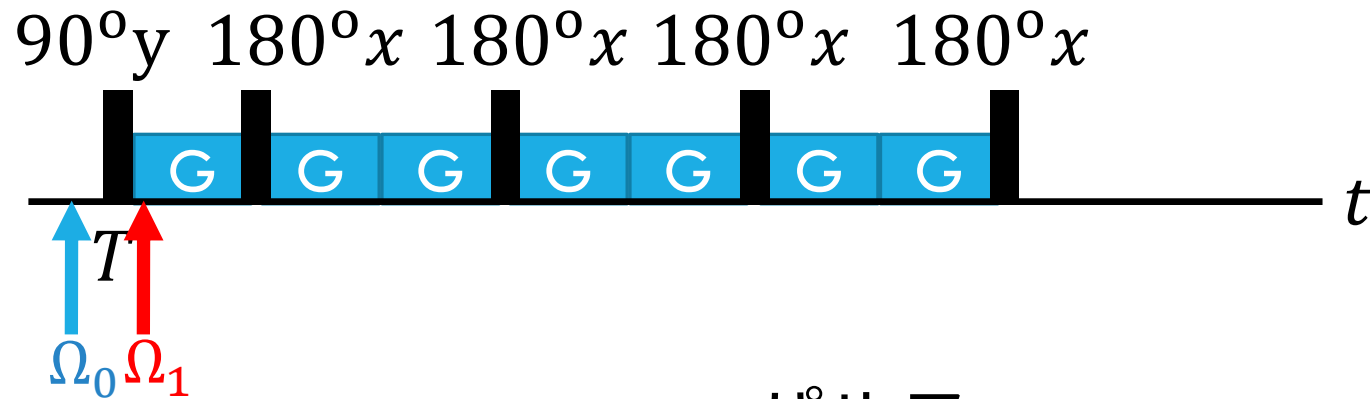


$$\Omega_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix} \leftarrow \begin{pmatrix} F_0 & F_1 \\ F_0^* & F_{-1}^* \\ Z_0 & Z_1 \end{pmatrix}$$





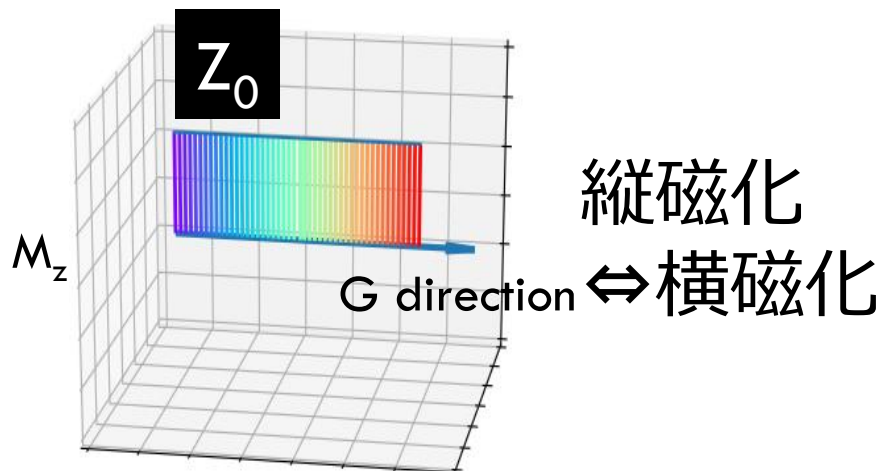
# CPMG (理想)



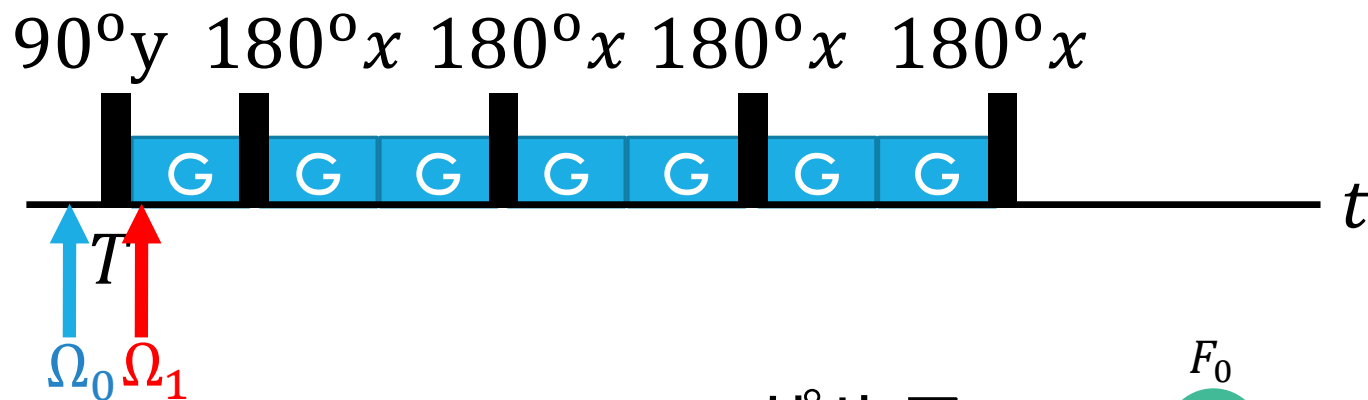
$$\Omega_0 = \begin{pmatrix} 0 & 0 \\ \uparrow & \downarrow \\ 0 & 0 \\ \downarrow & \uparrow \\ 1 & 0 \end{pmatrix}$$

RFパルス

$$\Omega_1 = T_{90y} \Omega_0$$



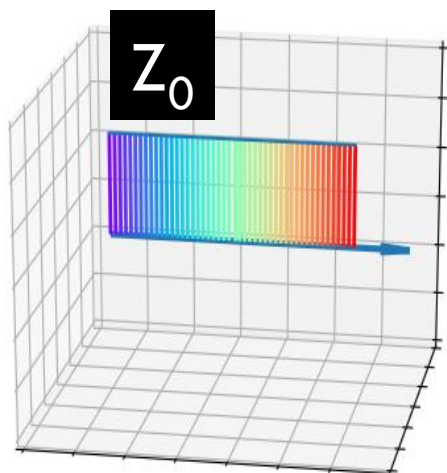
# CPMG (理想)



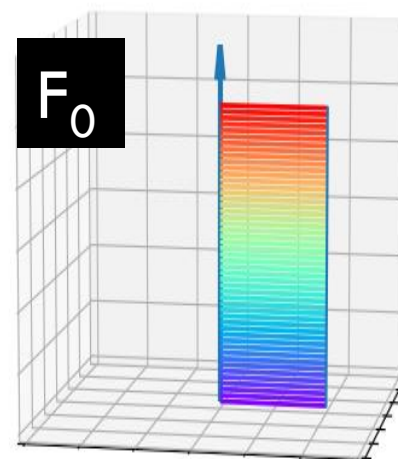
$$\Omega_0 = \begin{pmatrix} 0 & 0 \\ \uparrow & \downarrow \\ 0 & 0 \\ \downarrow & \uparrow \\ 1 & 0 \end{pmatrix}$$

RFパルス

$$\Omega_1 = T_{90y} \Omega_0 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$$

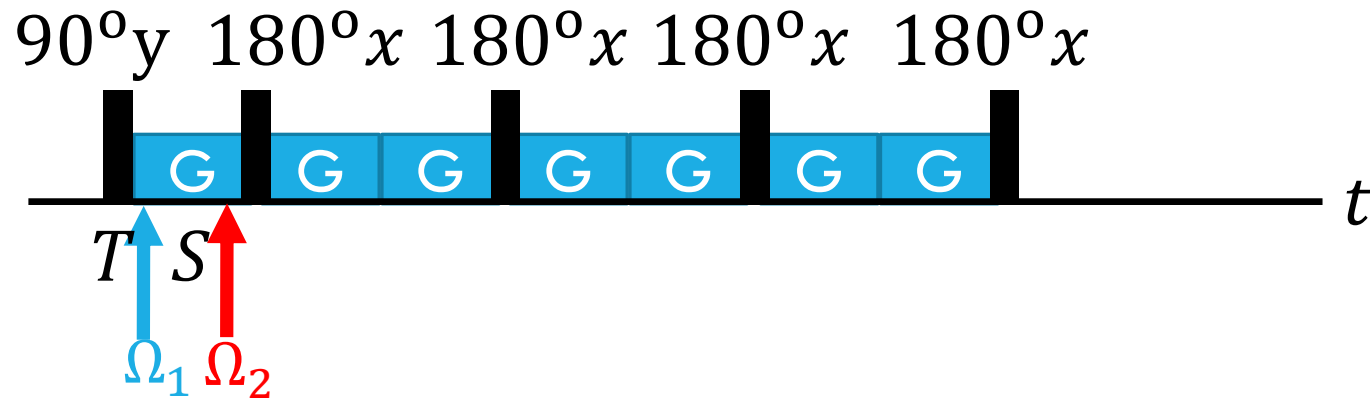


縦磁化  
→横磁化



FID発生

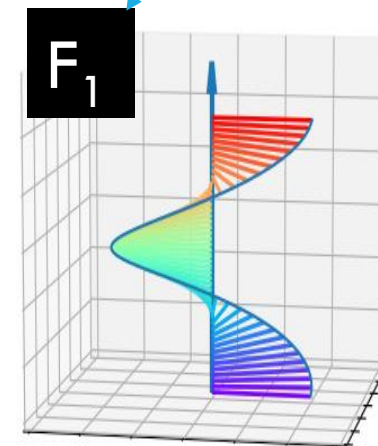
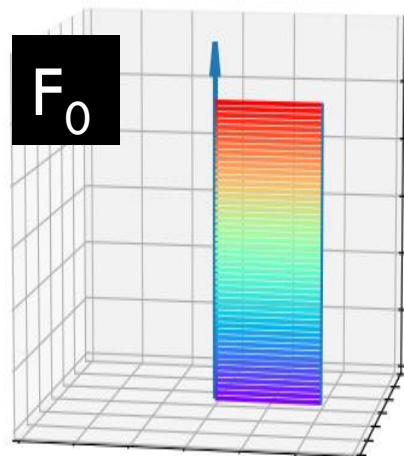
# CPMG (理想)



$$\Omega_1 = \begin{pmatrix} 1 & \rightarrow & 0 \\ * & \uparrow & 0 \\ 1 & \leftarrow & 0 \\ 0 & & 0 \end{pmatrix}$$

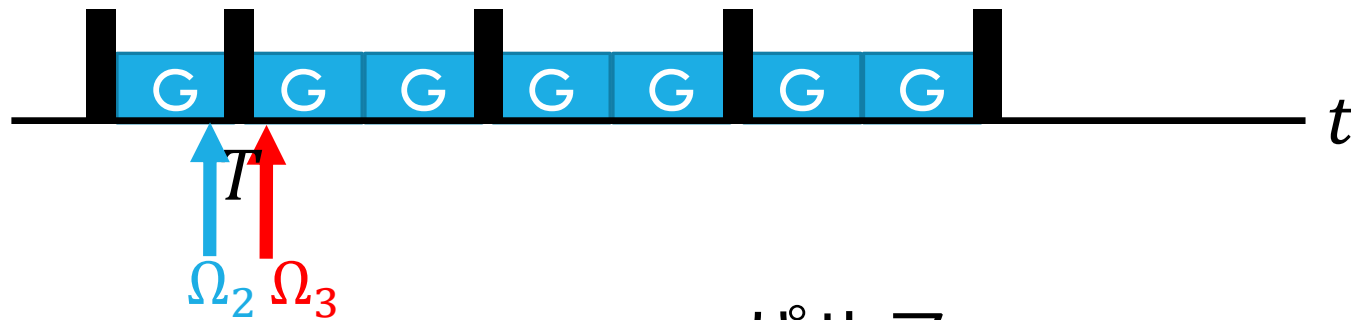
シフト演算子

$$\Omega_2 = S\Omega_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$



# CPMG (理想)

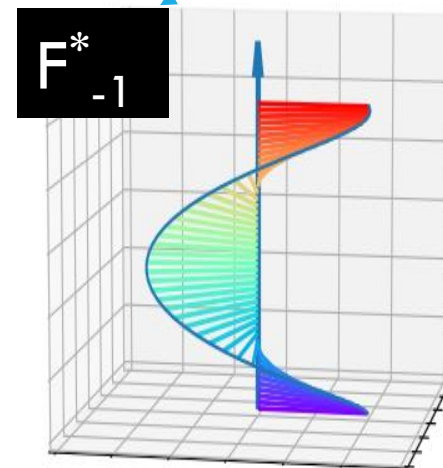
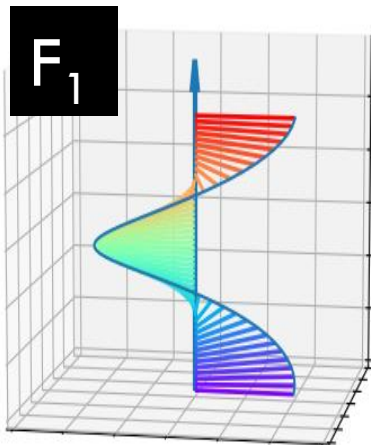
$90^\circ_y$   $180^\circ_x$   $180^\circ_x$   $180^\circ_x$   $180^\circ_x$



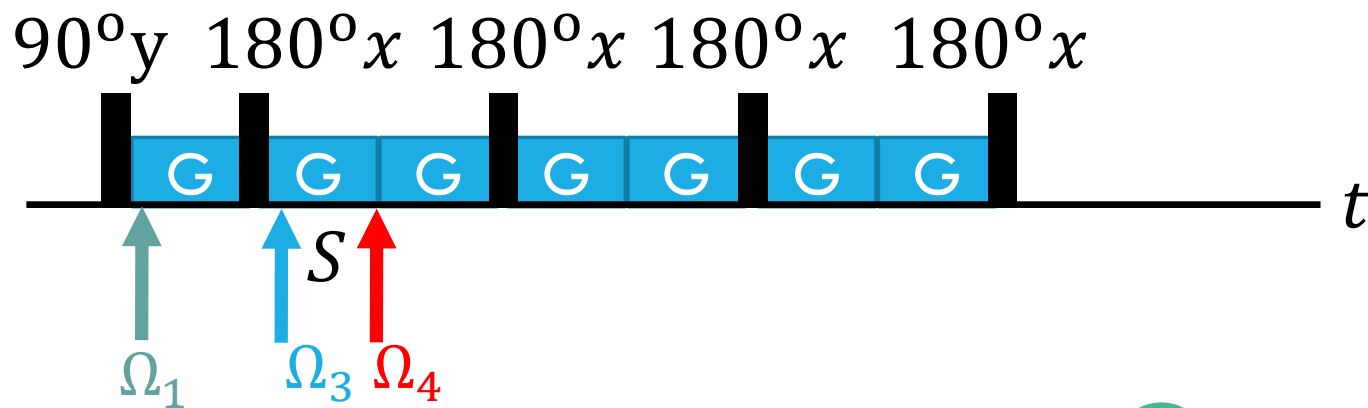
$$\Omega_2 = \begin{pmatrix} 0 & 1 \\ \uparrow & \downarrow \\ 0 & 0 \\ \downarrow & \uparrow \\ 0 & 0 \end{pmatrix}$$

RFパルス

$$\Omega_3 = T_{180x} \Omega_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$



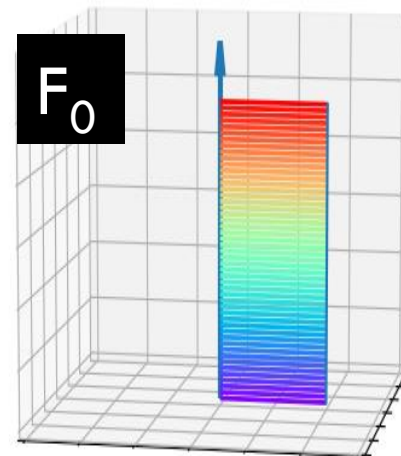
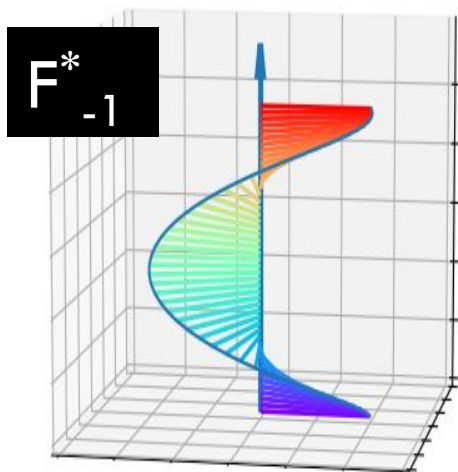
# CPMG (理想)



$$\Omega_3 = \begin{pmatrix} 0 & \rightarrow & 0 \\ * & \rightarrow & 1 \\ 0 & \leftarrow & 0 \\ 0 & & 0 \end{pmatrix}$$

シフト演算子

$$\Omega_4 = S\Omega_3 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$$



echo発生

$\Omega_1$ に戻る

# CPMG (理想)

$90^\circ_y$   $180^\circ_x$   $180^\circ_x$   $180^\circ_x$   $180^\circ_x$



演算子

$T$   $S$   $T$   $S$   $S$   $T$   $S$   $S$   $T$   $S$   $S$   $T$

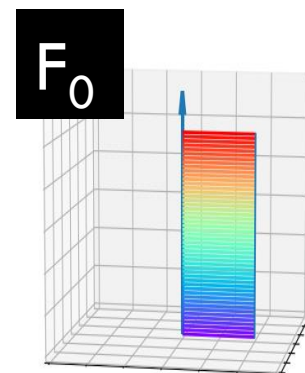
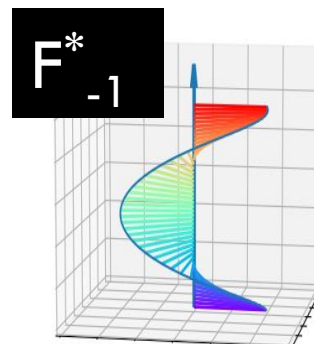
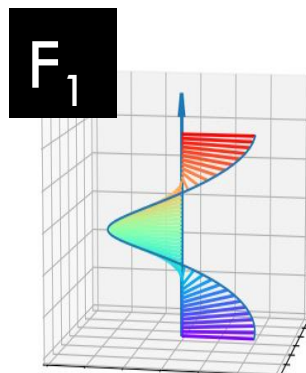
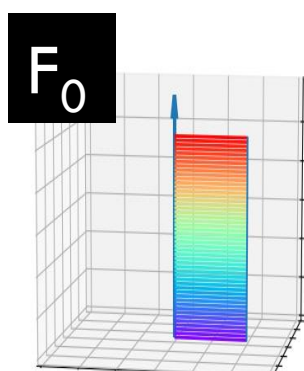
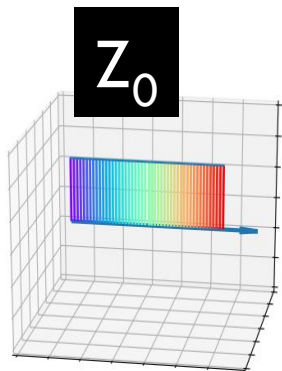
$$\Omega_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\Omega_1 = T_{90y} \Omega_0 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Omega_2 = S \Omega_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Omega_3 = T_{180x} \Omega_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

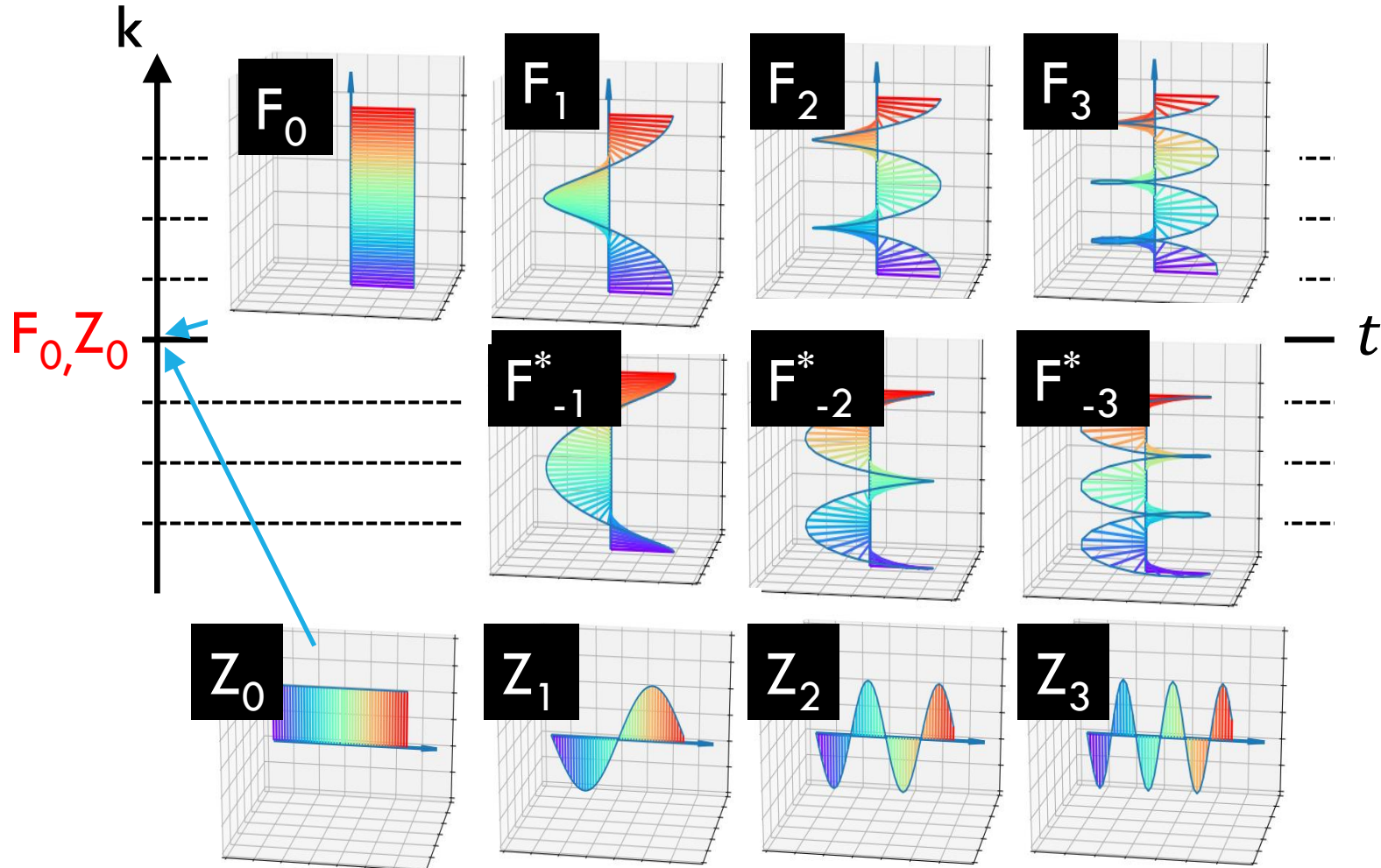
$$\Omega_4 = S \Omega_3 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$$



“グラフ”にしてみる

## EPG-GRAPHING

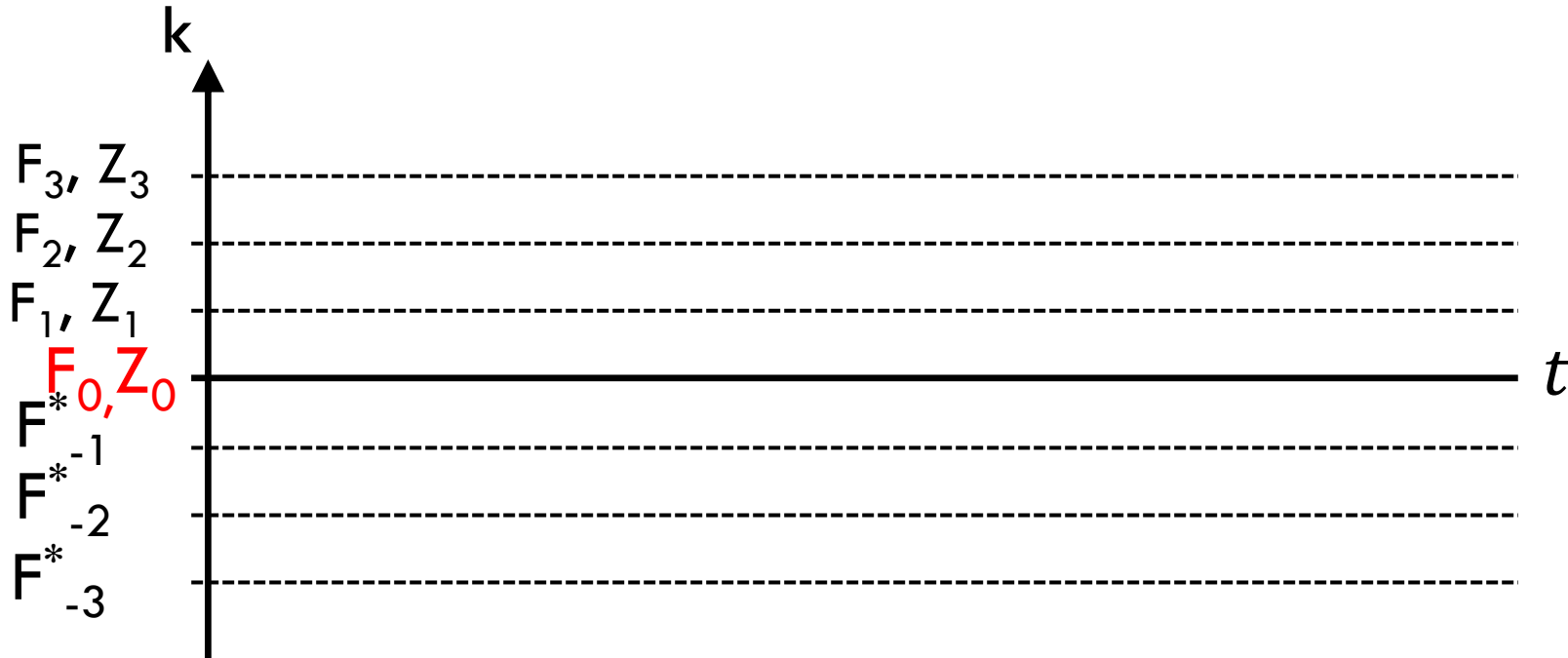
縦軸は  $k$  or “states”





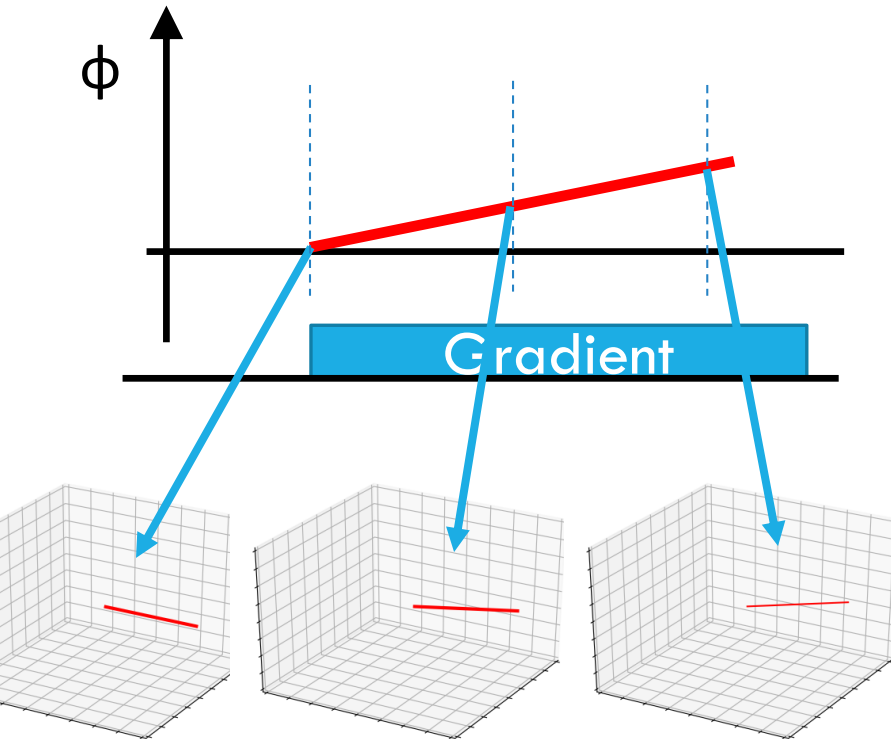
# EPG-GRAPHING

縦軸は  $k$  or “states”



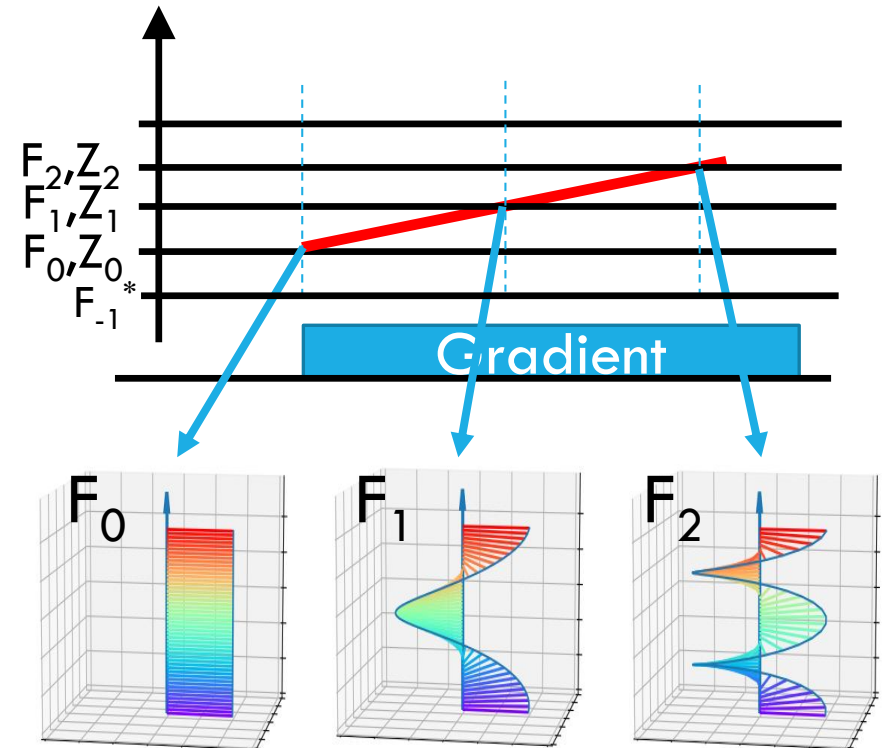
# 古典的PGとEPGの違い

## 古典的PG



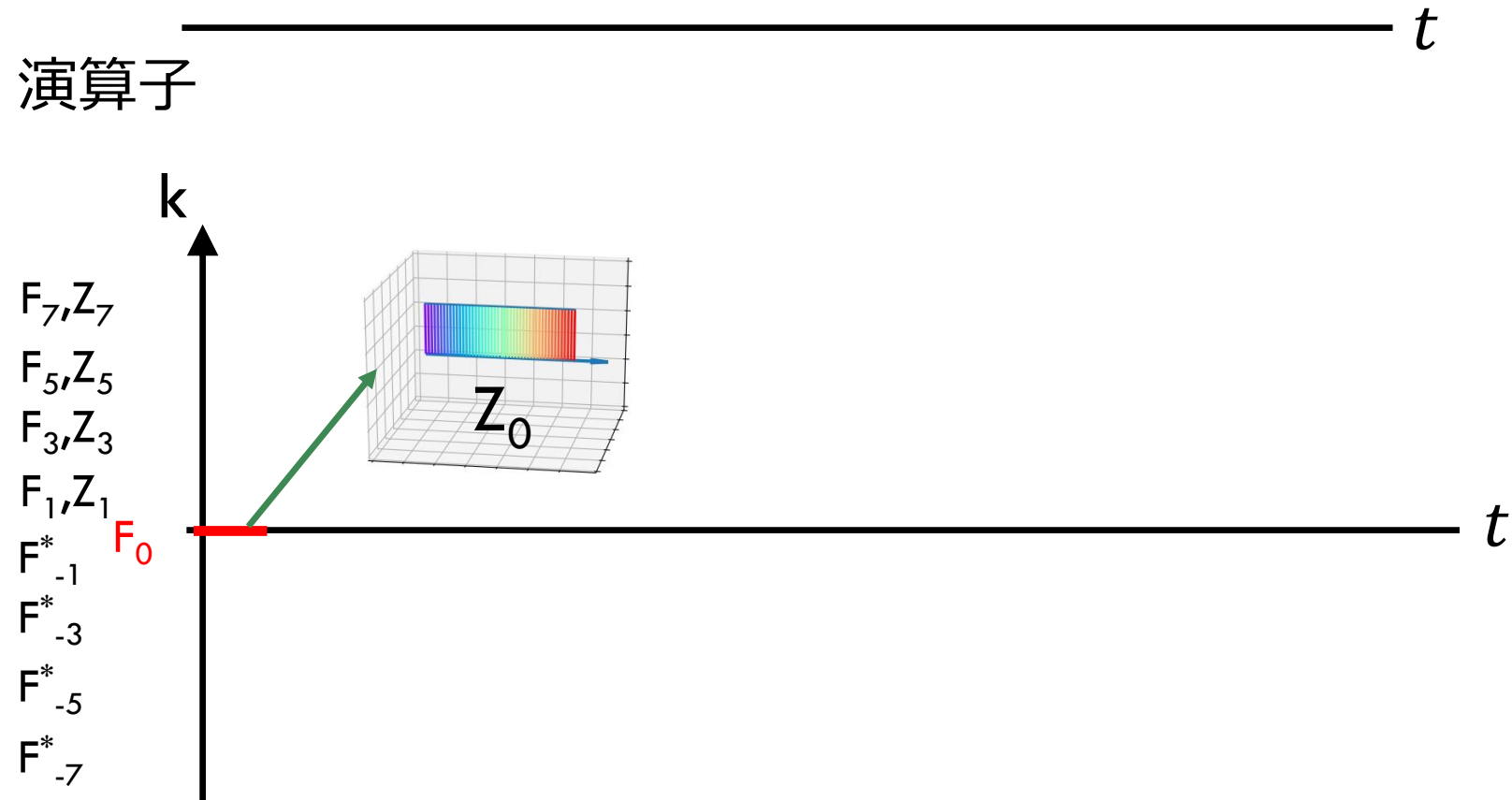
- 縦軸は横磁化の位相
- 単一スピ  
(spin isochromat)

## k or "states" EPG

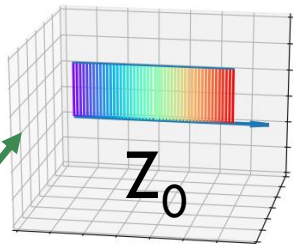


- 縦軸はk or "states"
- 多数スピ  
(spin isochromats)

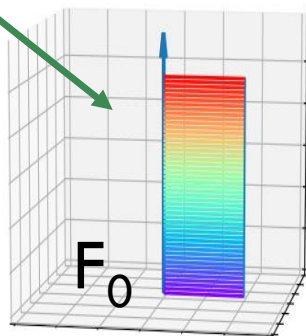
## EPG – CPMGシーケンス



## EPG – CPMGシーケンス

 $90^\circ_y$  $t$ 演算子  $T$  $k$  $F_7, Z_7$  $F_5, Z_5$  $F_3, Z_3$  $F_1, Z_1$  $F_0$  $F_{-1}^*$  $F_{-3}^*$  $F_{-5}^*$  $F_{-7}^*$  $Z_0$ RF90<sub>y</sub>

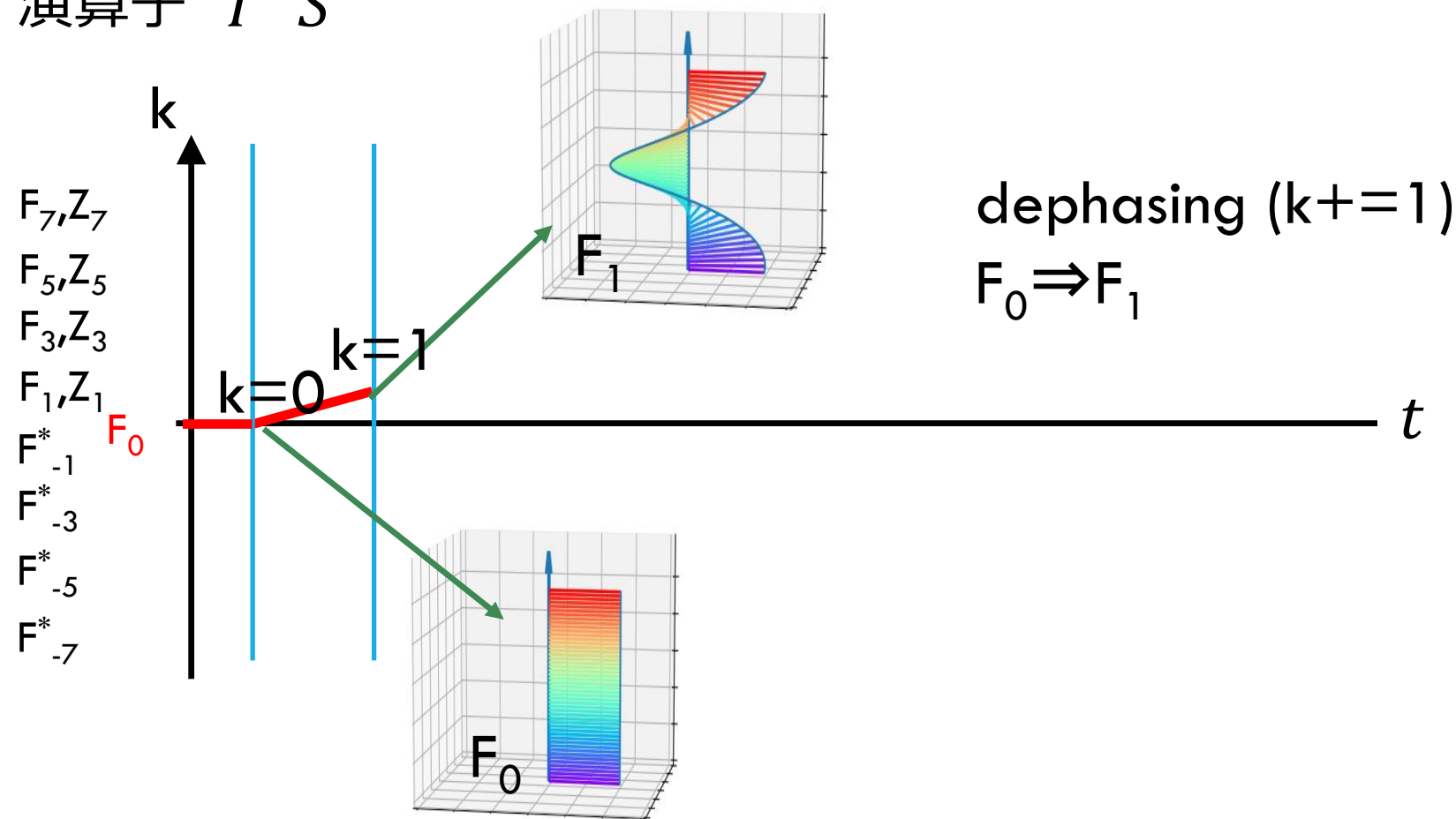
縦磁化⇒横磁化

 $t$  $F_0$

# EPG – CPMGシーケンス



演算子  $T$   $S$

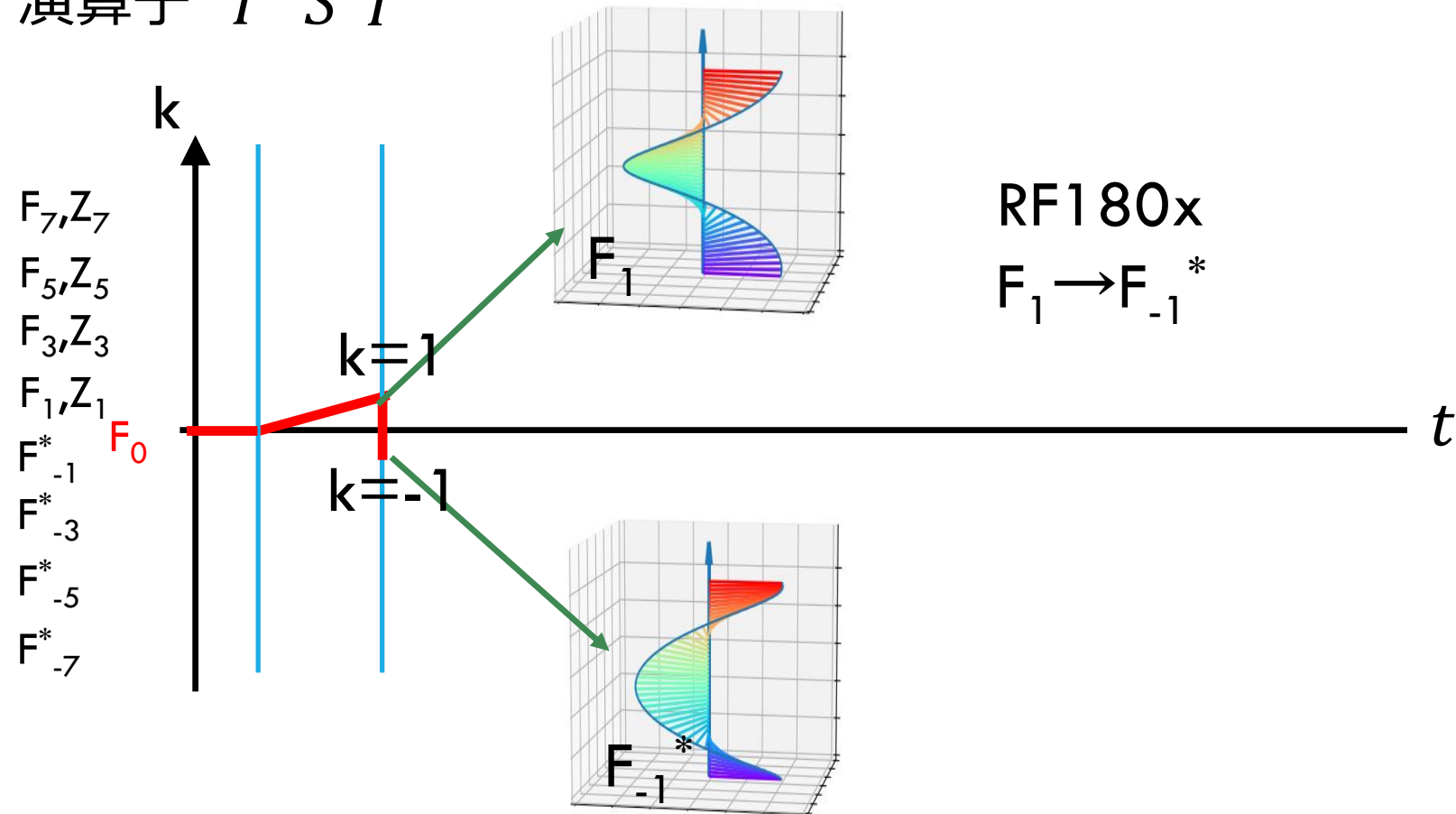


# EPG – CPMGシーケンス

$90^\circ_y$   $180^\circ_x$



演算子  $T$   $S$   $T$

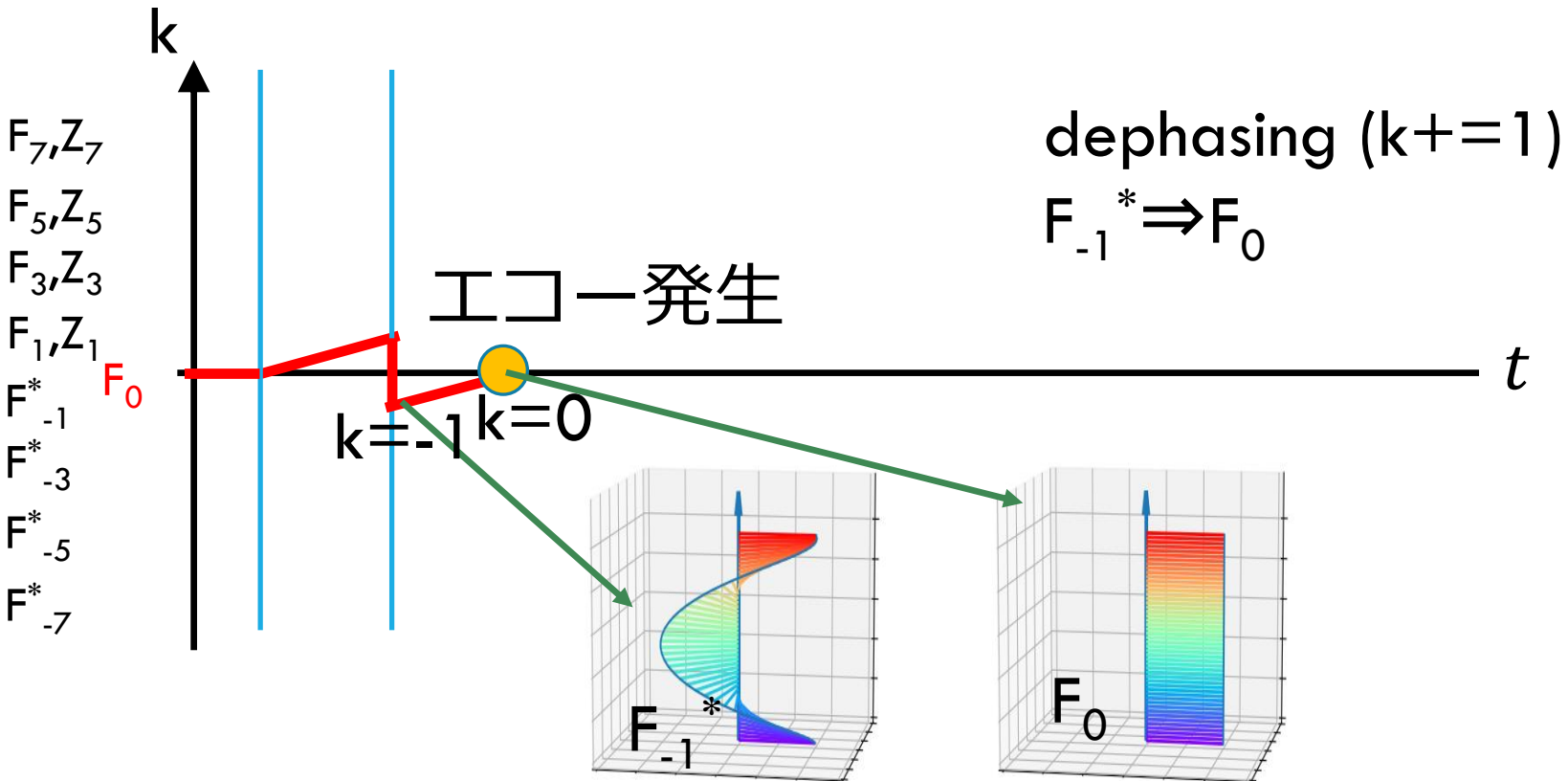


# EPG – CPMGシーケンス

$90^\circ_y$   $180^\circ_x$



演算子  $T$   $S$   $T$   $S$

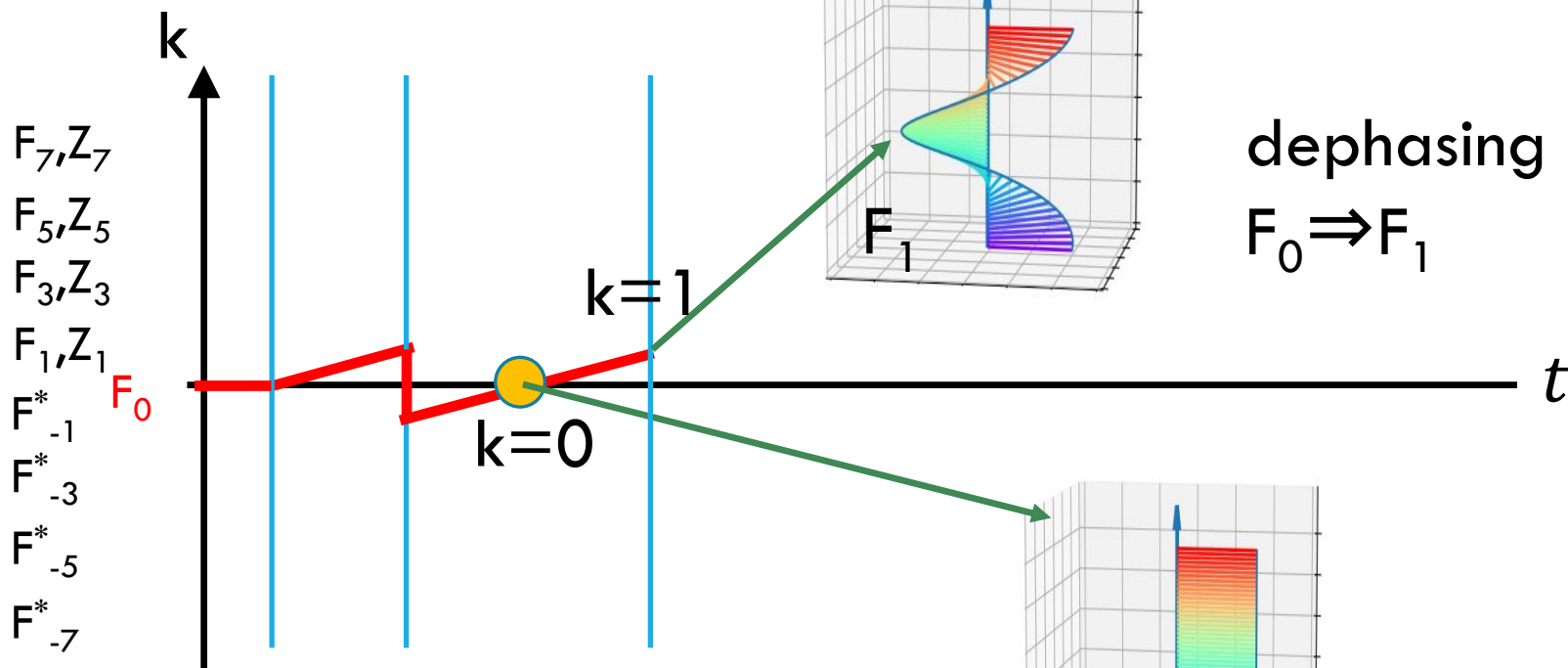


# EPG – CPMGシーケンス

$90^\circ_y$   $180^\circ_x$



演算子  $T$   $S$   $T$   $S$   $S$



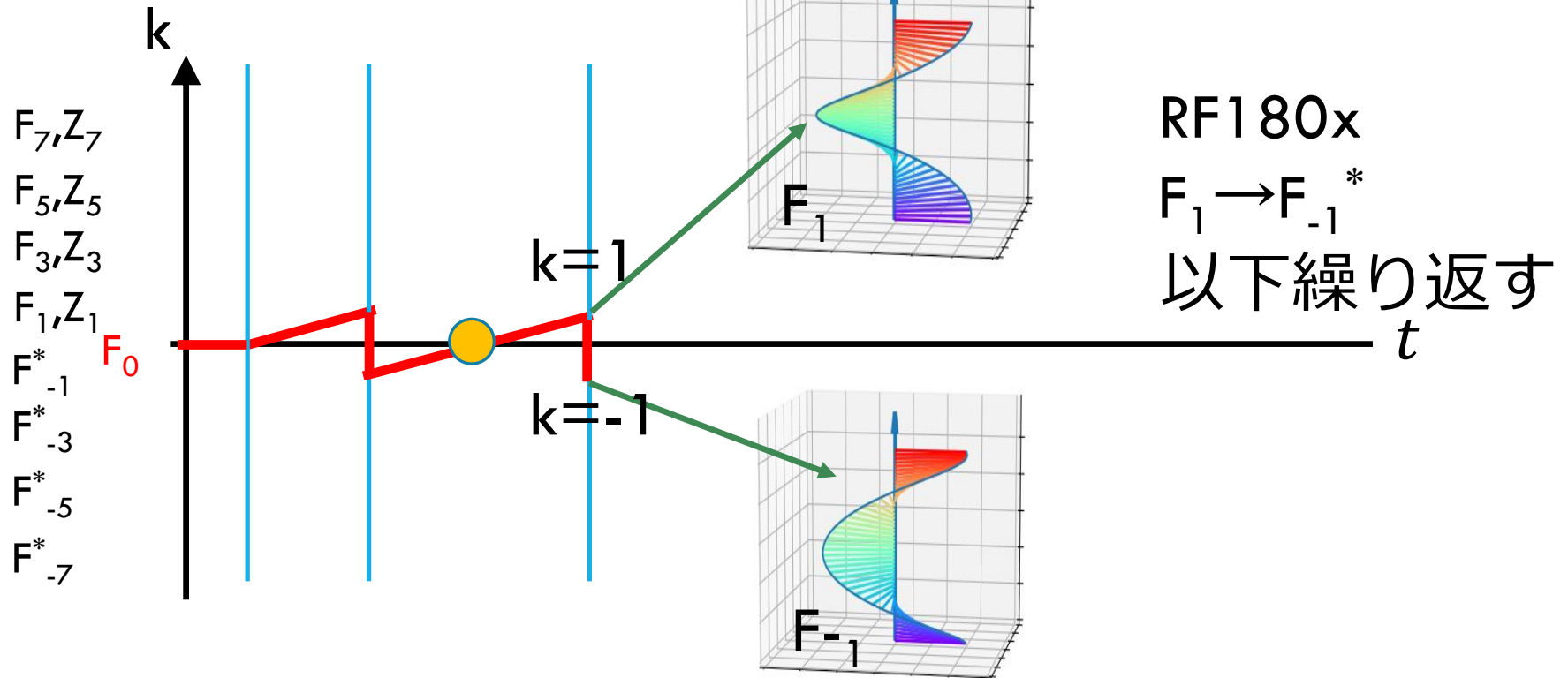


# EPG – CPMGシーケンス

$90^\circ_y$   $180^\circ_x$   $180^\circ_x$



演算子  $T$   $S$   $T$   $S$   $S$   $T$

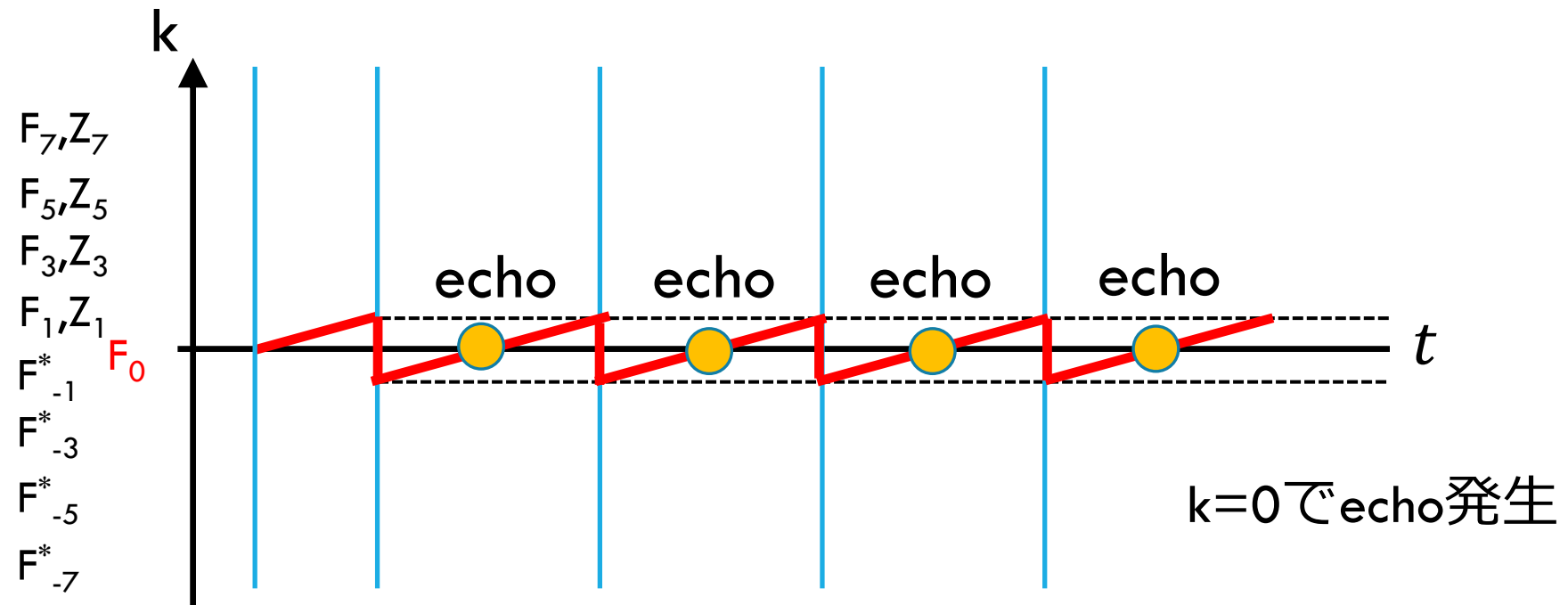


# EPG – CPMGシーケンス

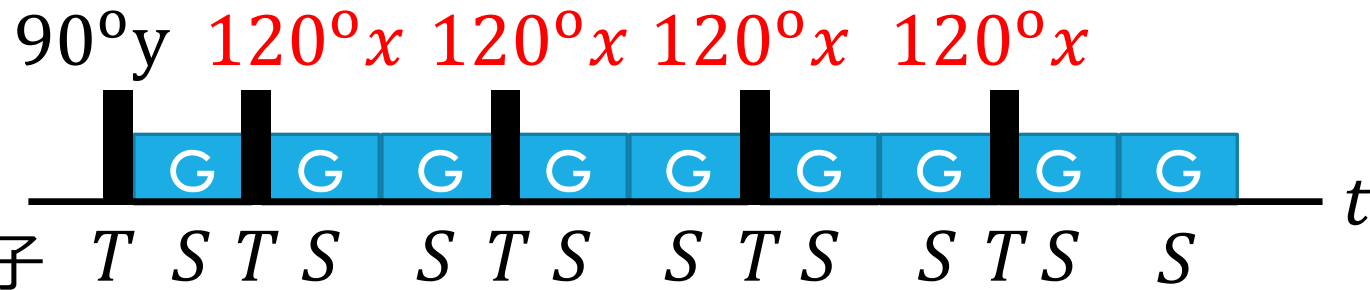
$90^\circ_y$   $180^\circ_x$   $180^\circ_x$   $180^\circ_x$   $180^\circ_x$



演算子  $T$   $S$   $T$   $S$   $S$   $T$   $S$   $S$   $T$   $S$   $S$   $T$   $S$   $S$



# MULTI ECHO



• RFパルス

$$T_{90y} = \begin{pmatrix} 0.5 & -0.5 & 1 \\ -0.5 & 0.5 & 1 \\ -0.5 & -0.5 & 0 \end{pmatrix}$$

• シフト演算子

$$S(+1): F_k \rightarrow F_{k+1}$$

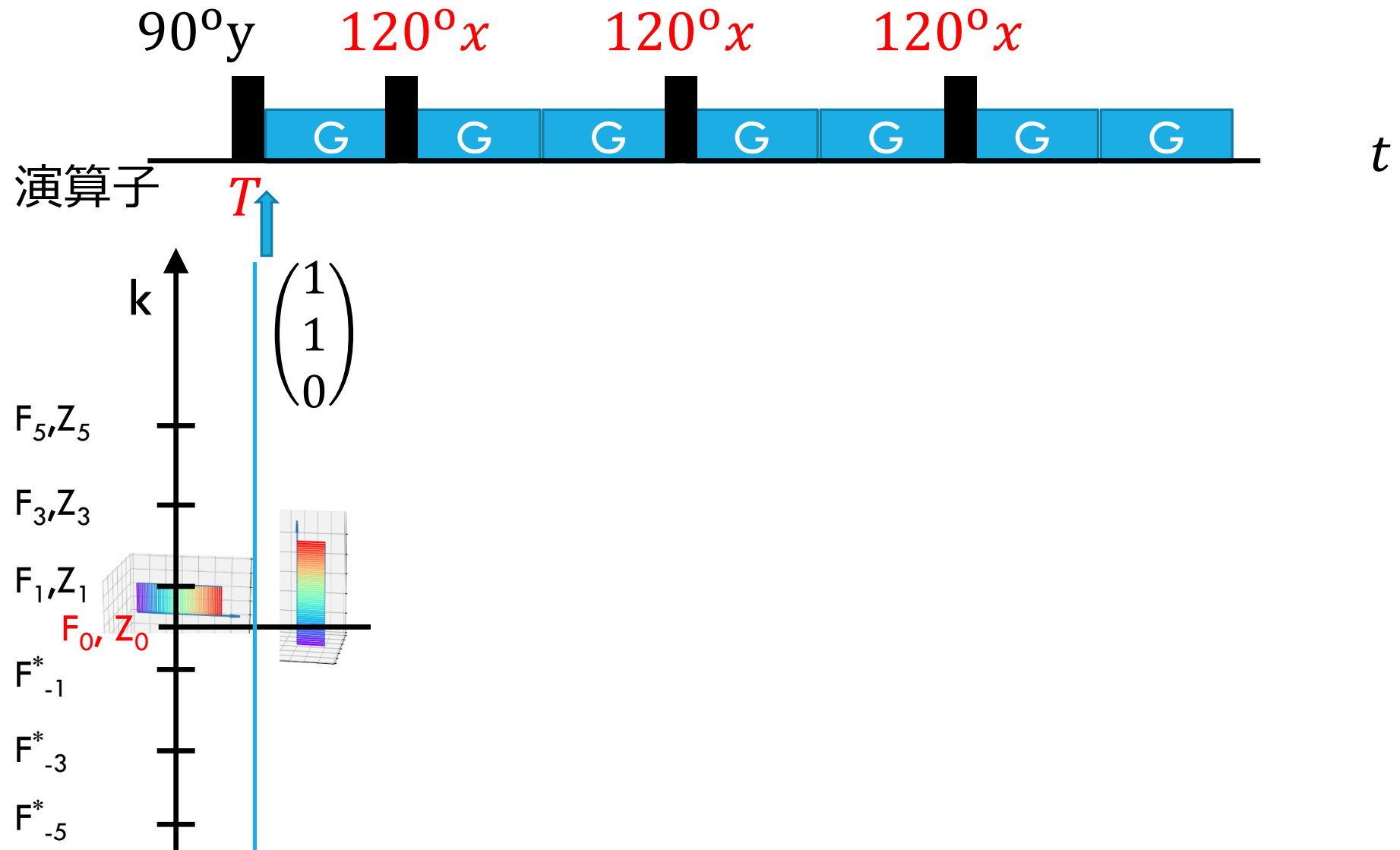
$$Z_k \rightarrow Z_k$$

$$T_{120x}$$

$$= \begin{pmatrix} 0.25 & 0.75 & 0 \\ 0.75 & 0.25 & 0 \\ -0.43i & 0.43i & -0.5 \end{pmatrix}$$

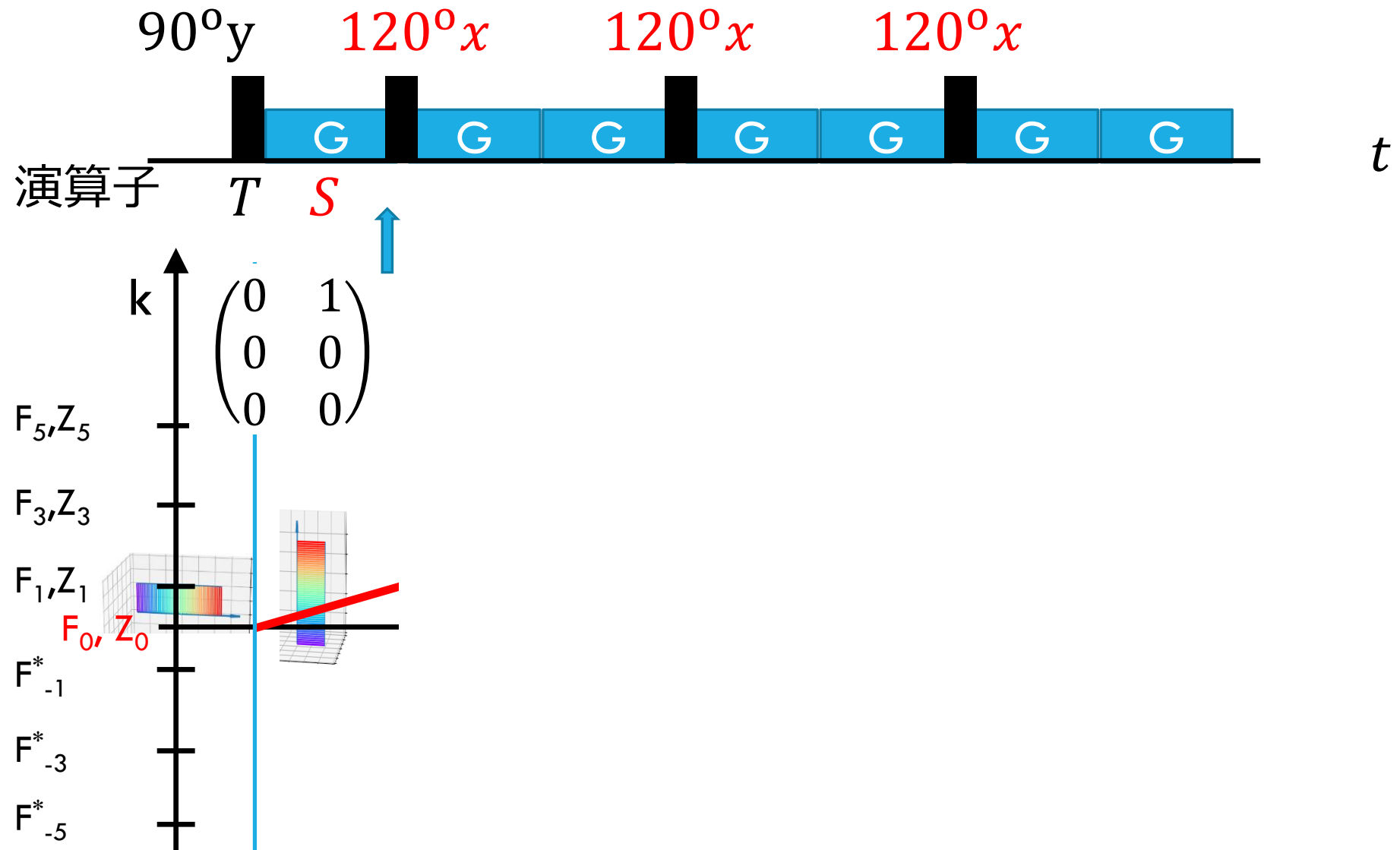
• 緩和項は無視

# MULTI ECHO



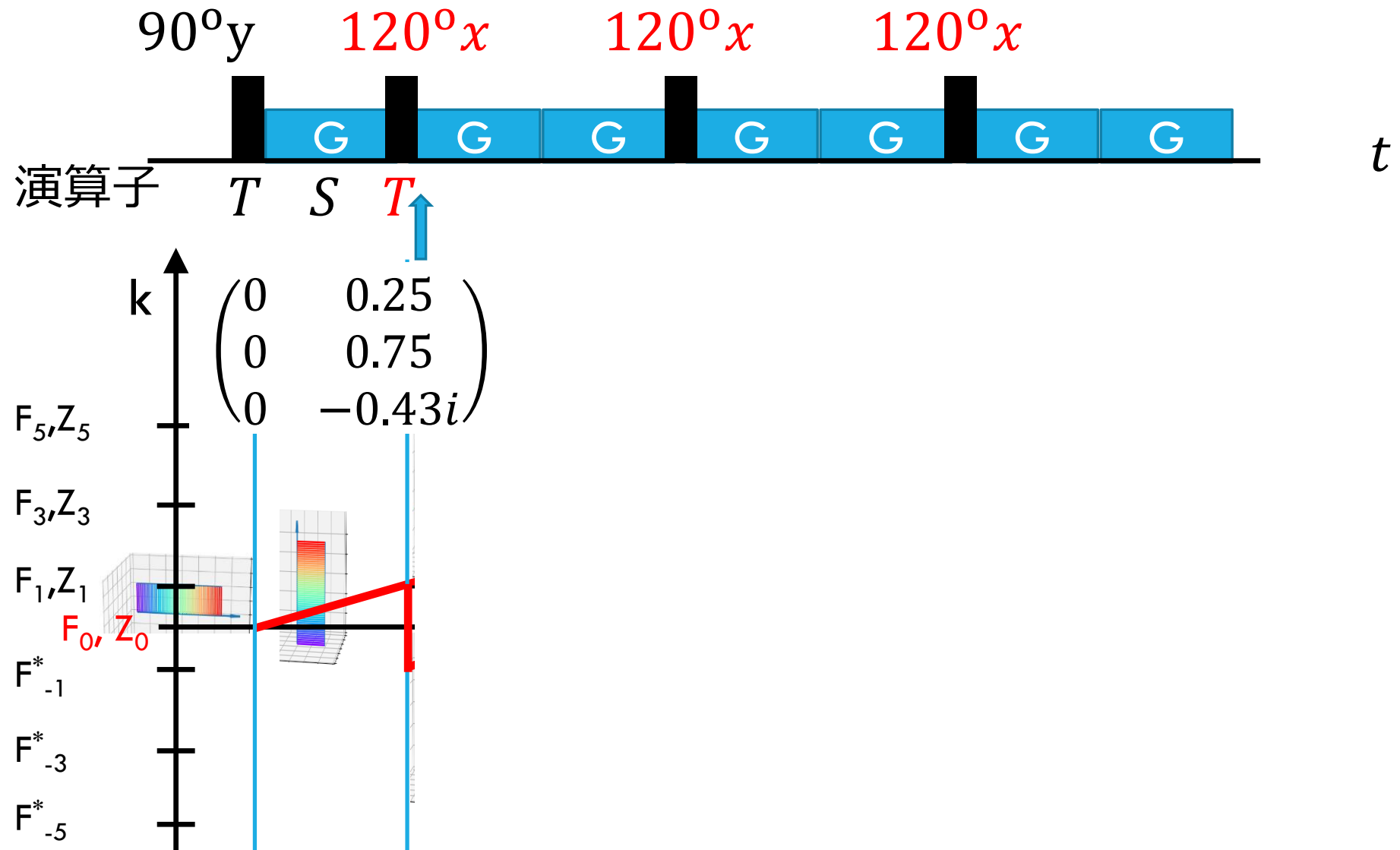
- エコ強度：行列計算で簡単に求められる

# MULTI ECHO



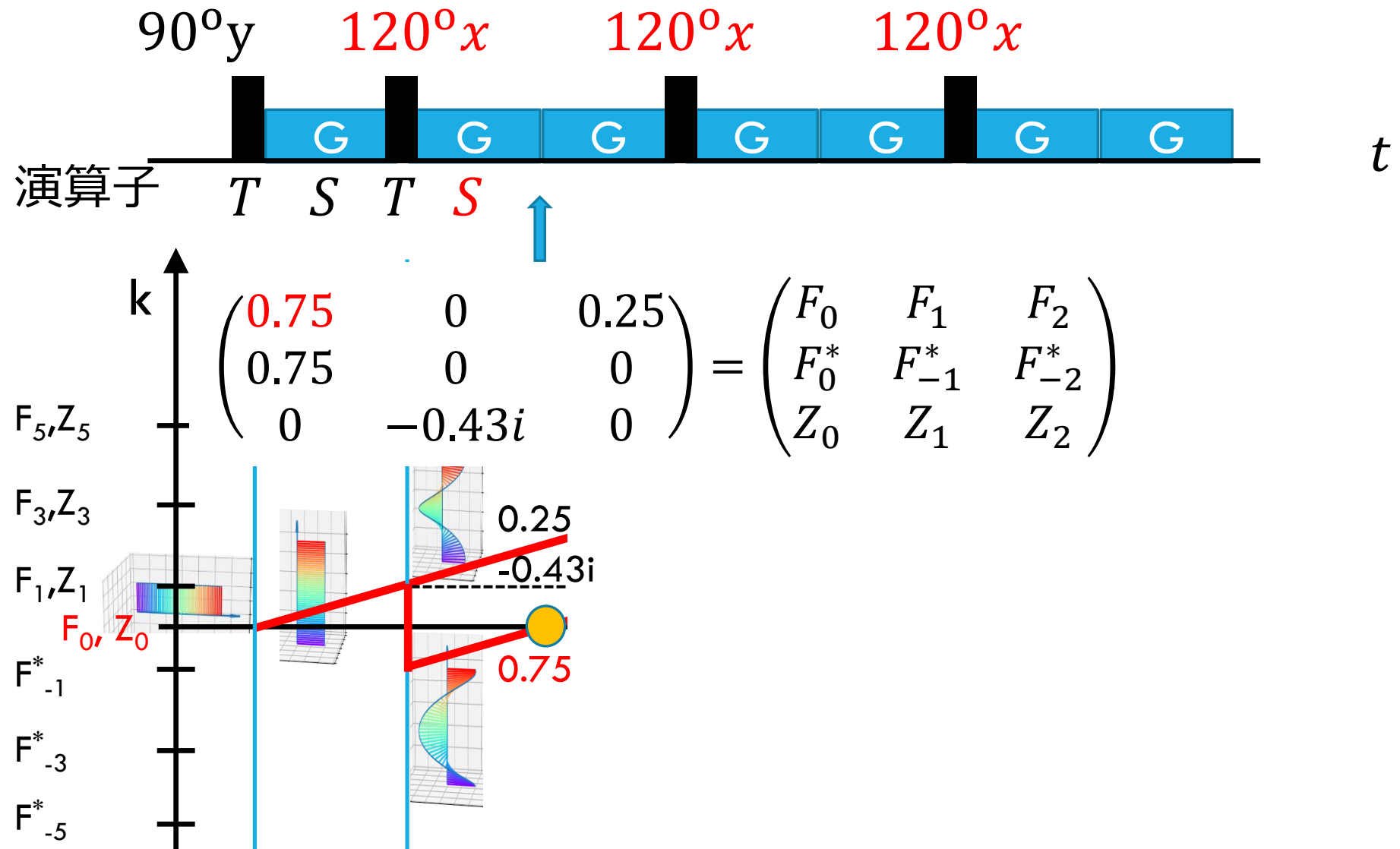
- エコー強度：行列計算で簡単に求められる

# MULTI ECHO



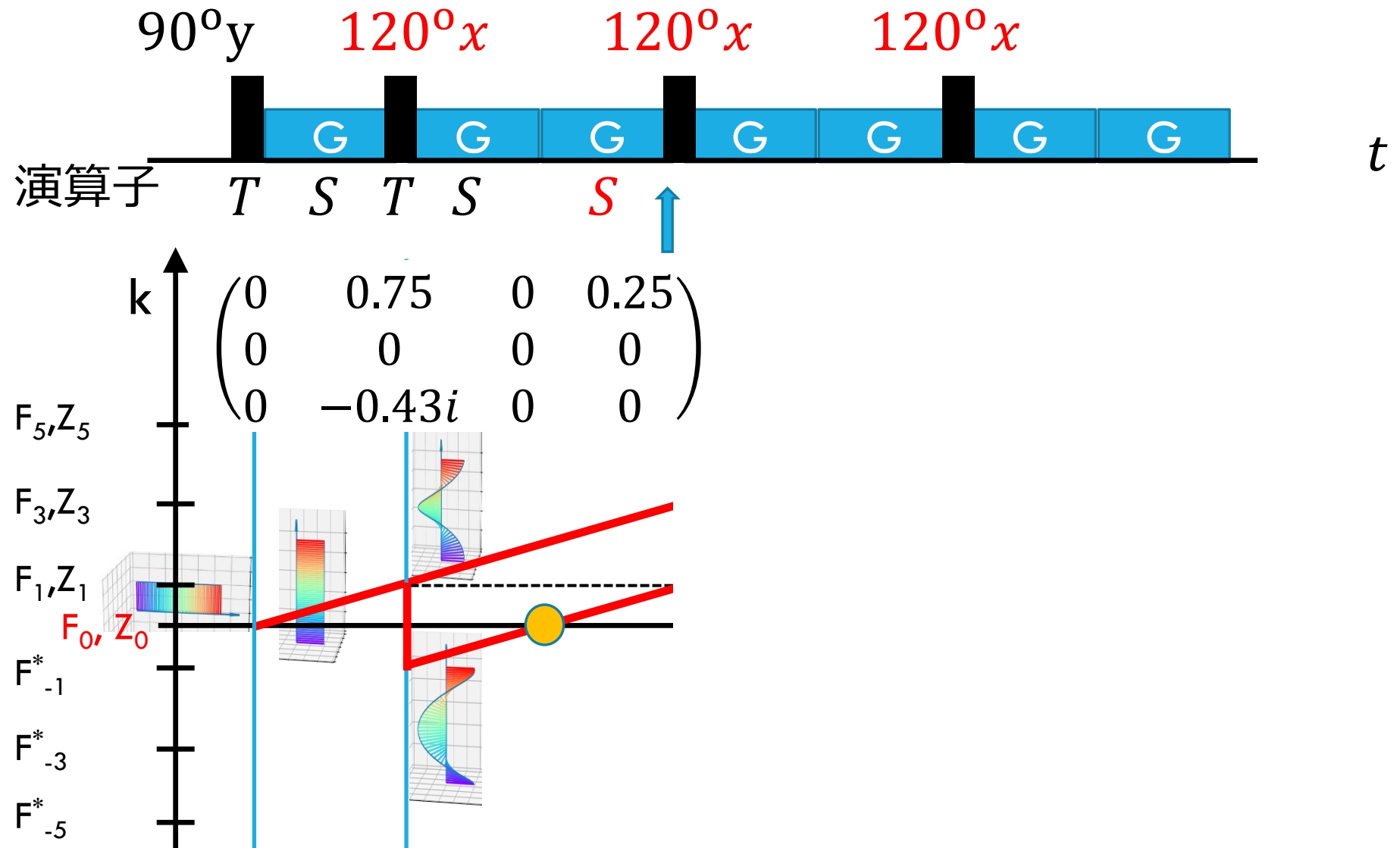
- エコー強度：行列計算で簡単に求められる

# MULTI ECHO



- エコー強度：行列計算で簡単に求められる

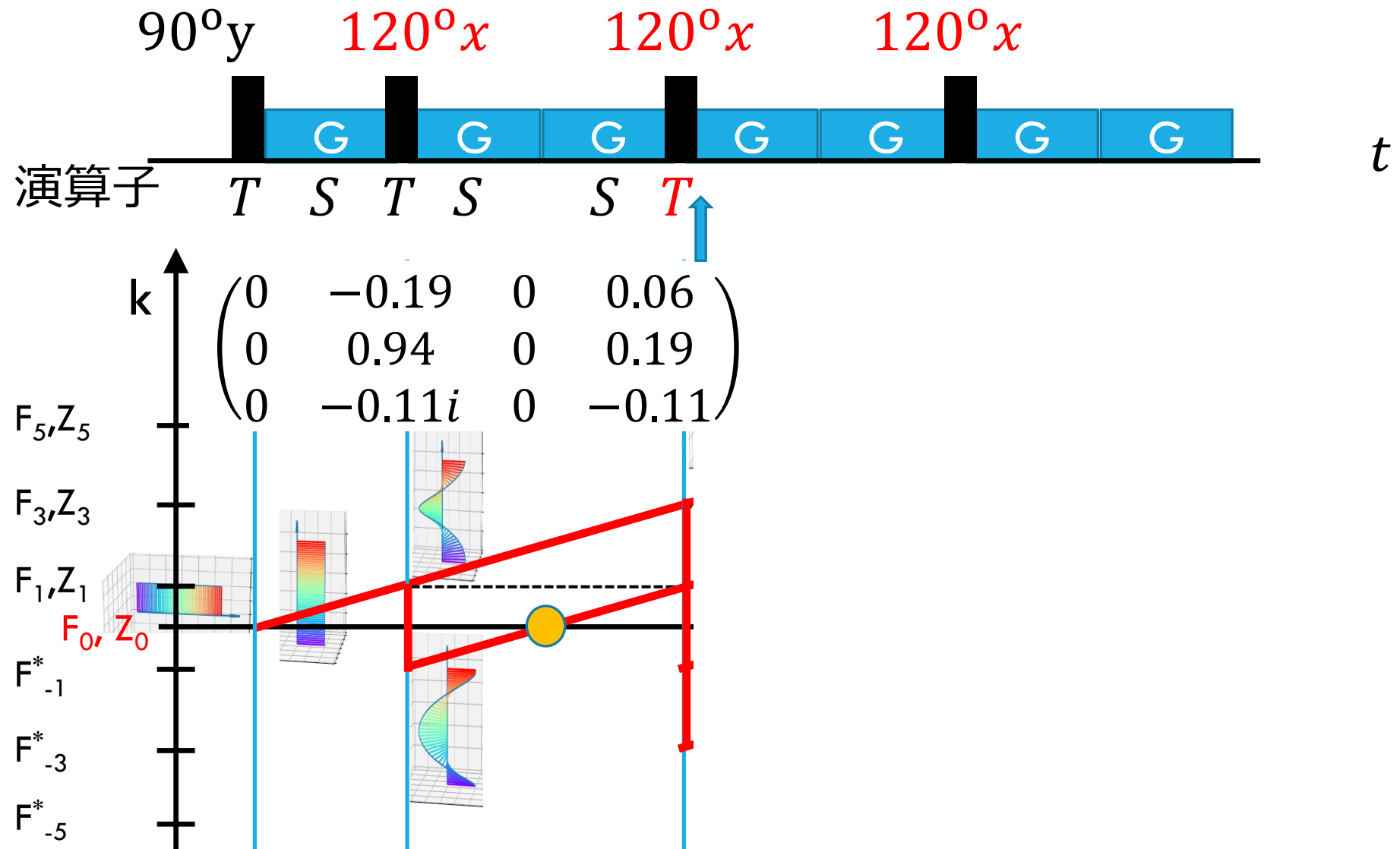
# MULTI ECHO



- エコー強度：行列計算で簡単に求められる

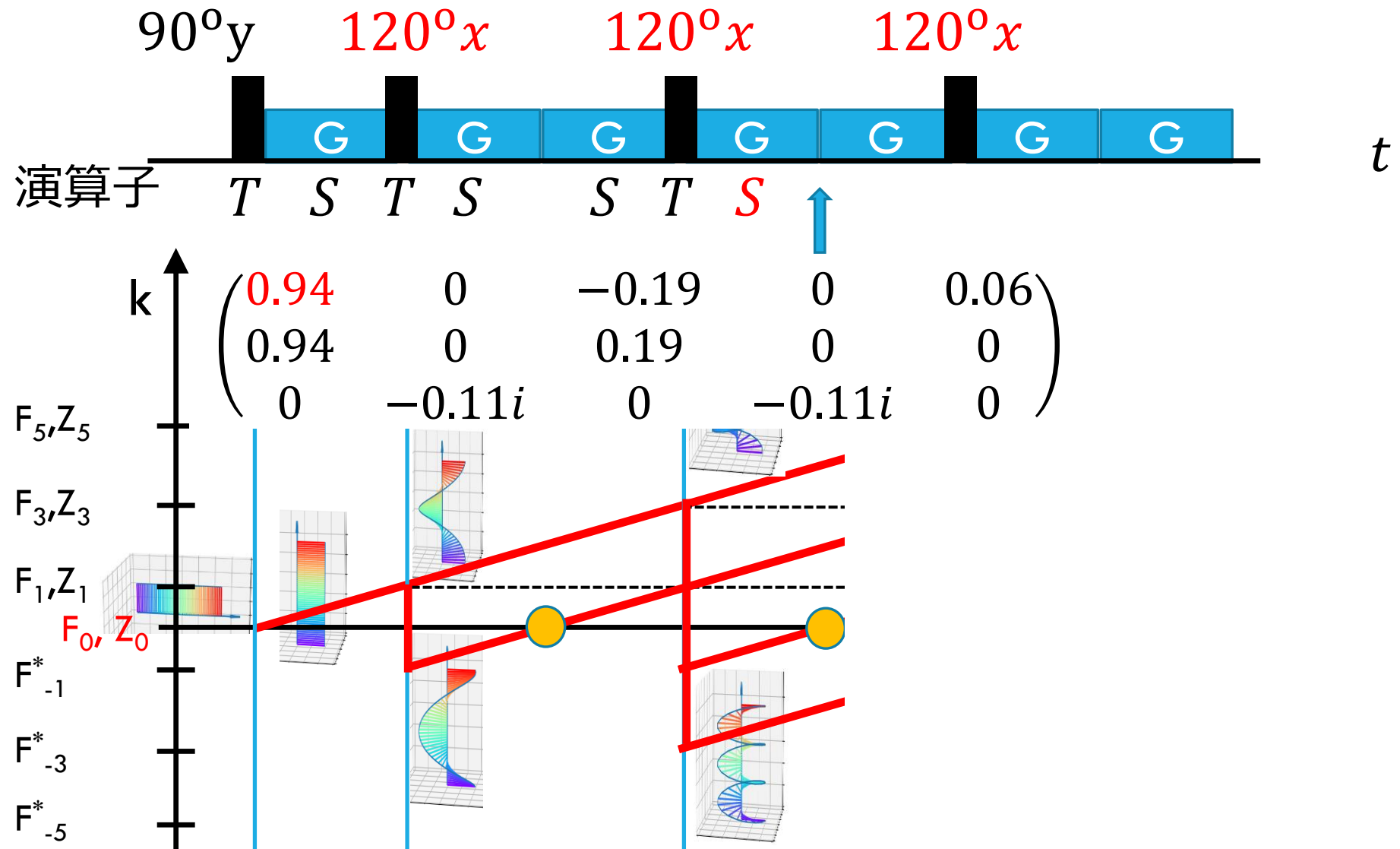


# MULTI ECHO



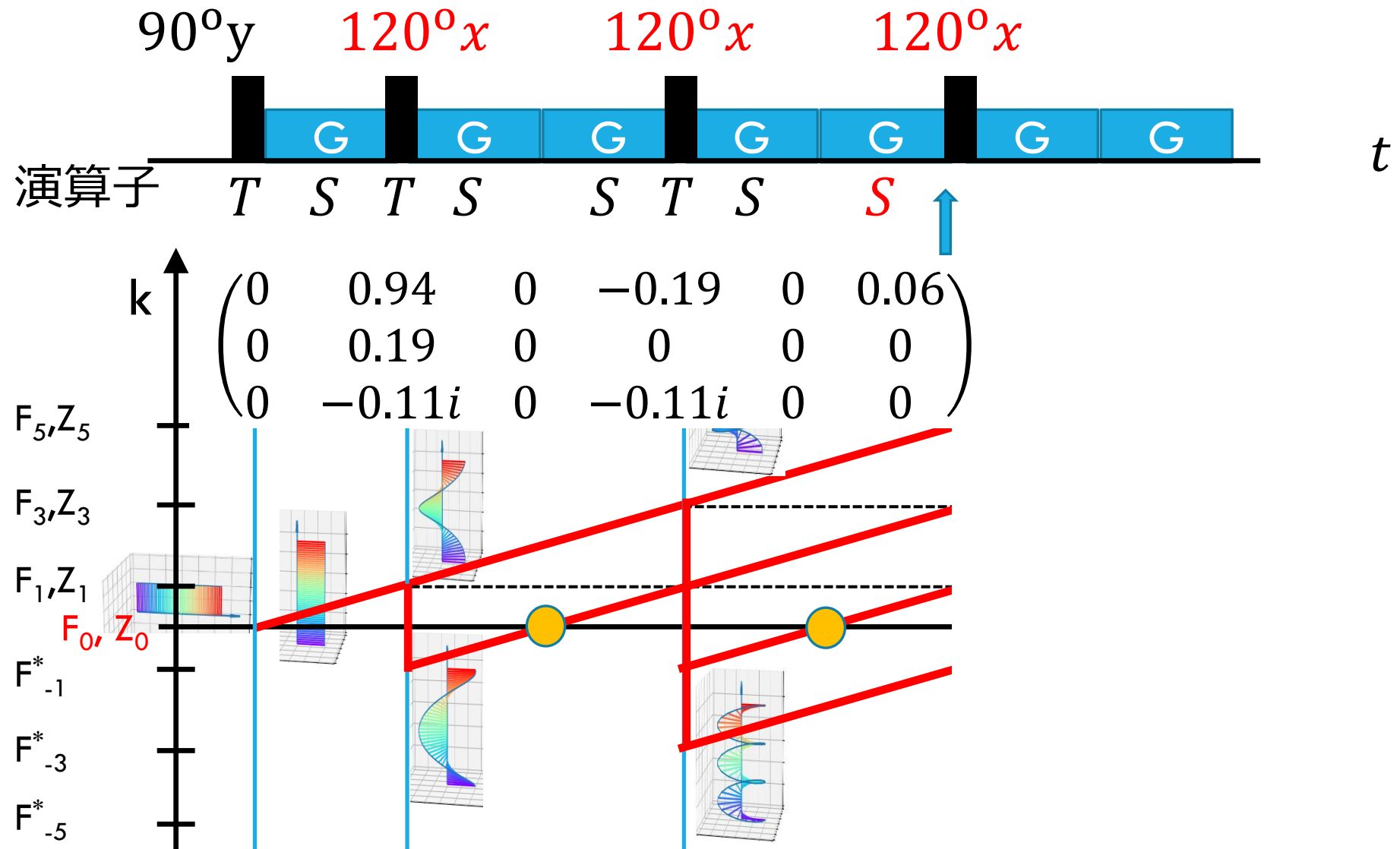
- エコー強度：行列計算で簡単に求められる

# MULTI ECHO



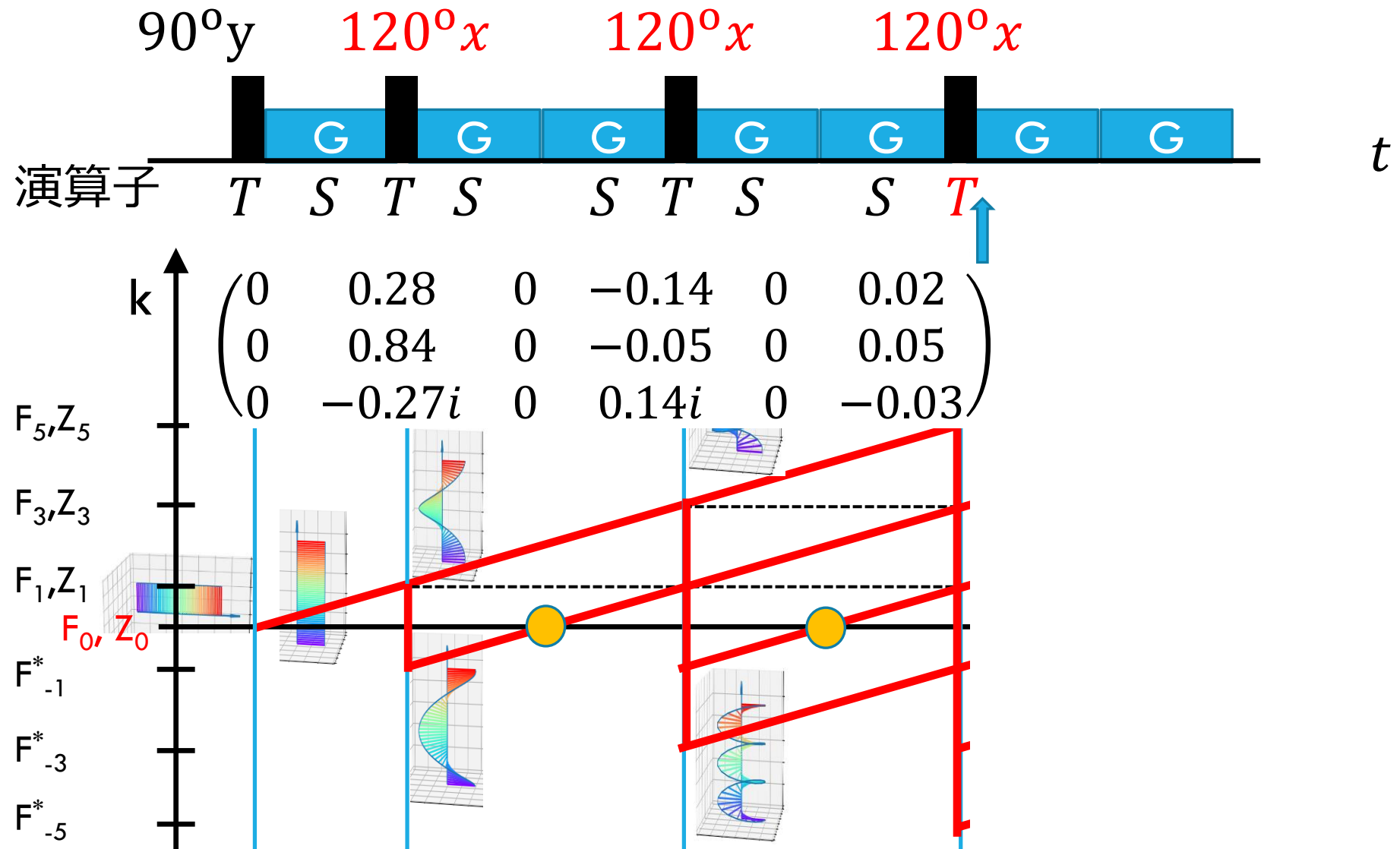
- エコー強度：行列計算で簡単に求められる

## MULTI ECHO



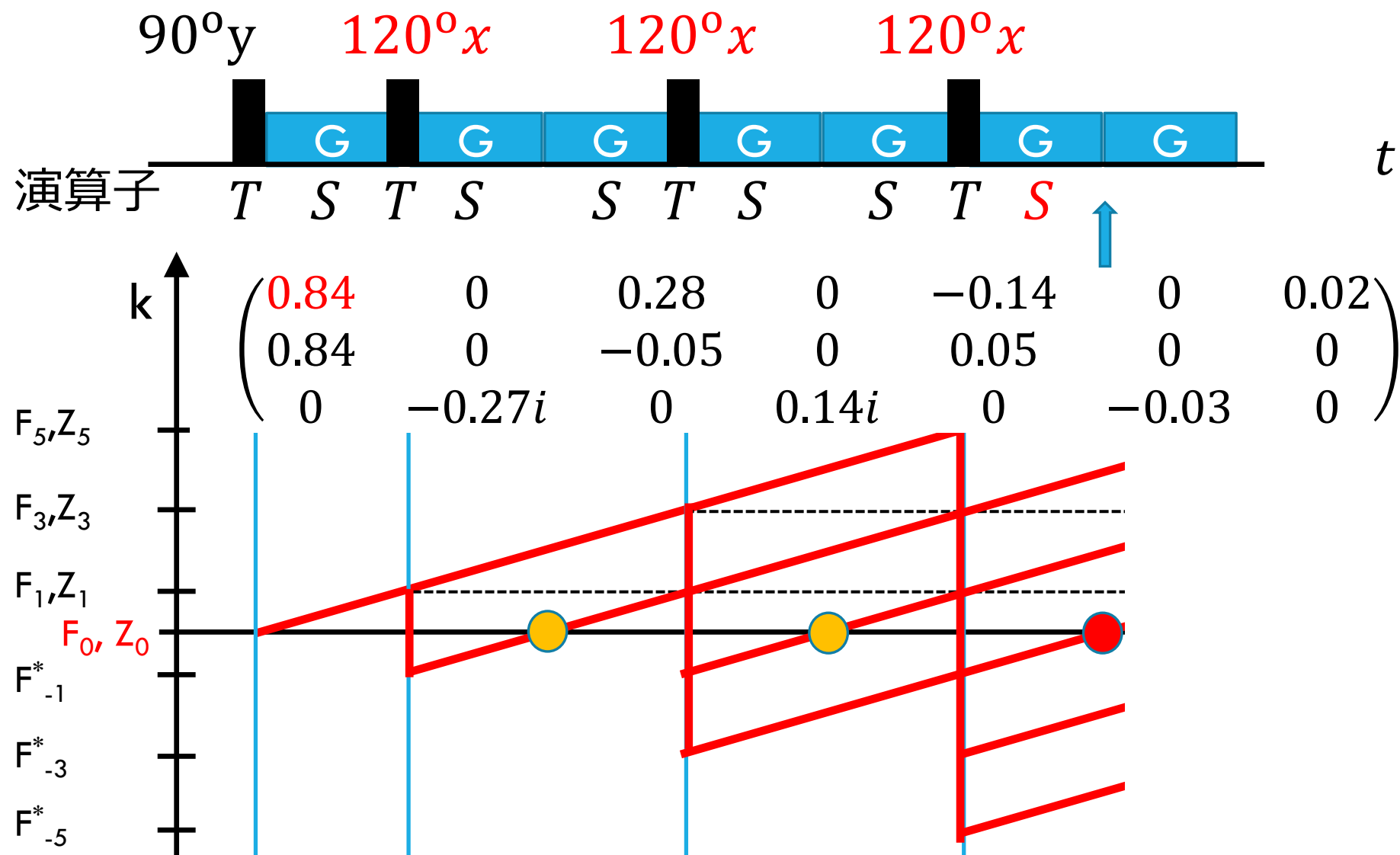
- エコー強度：行列計算で簡単に求められる

# MULTI ECHO



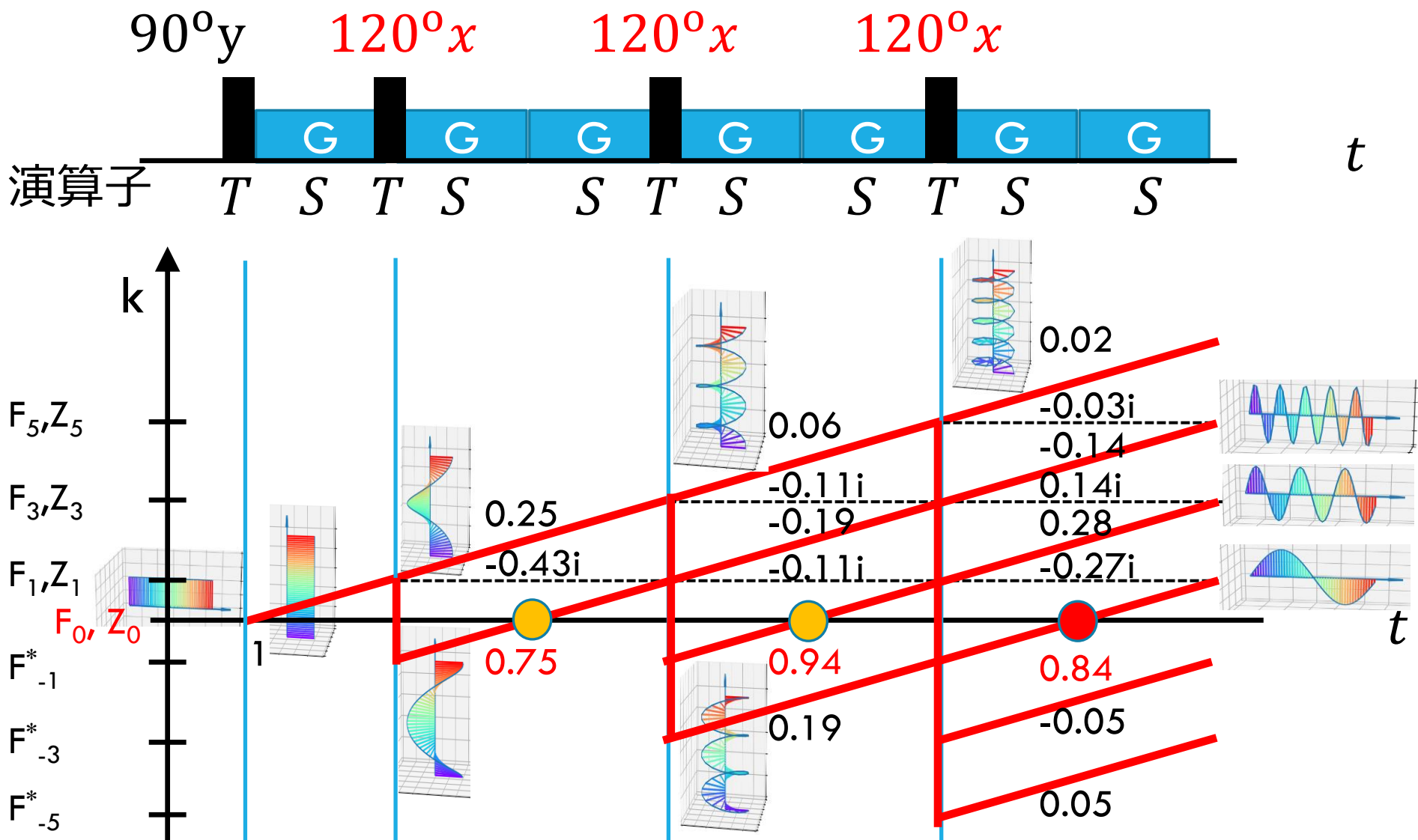
- エコー強度：行列計算で簡単に求められる

## MULTI ECHO



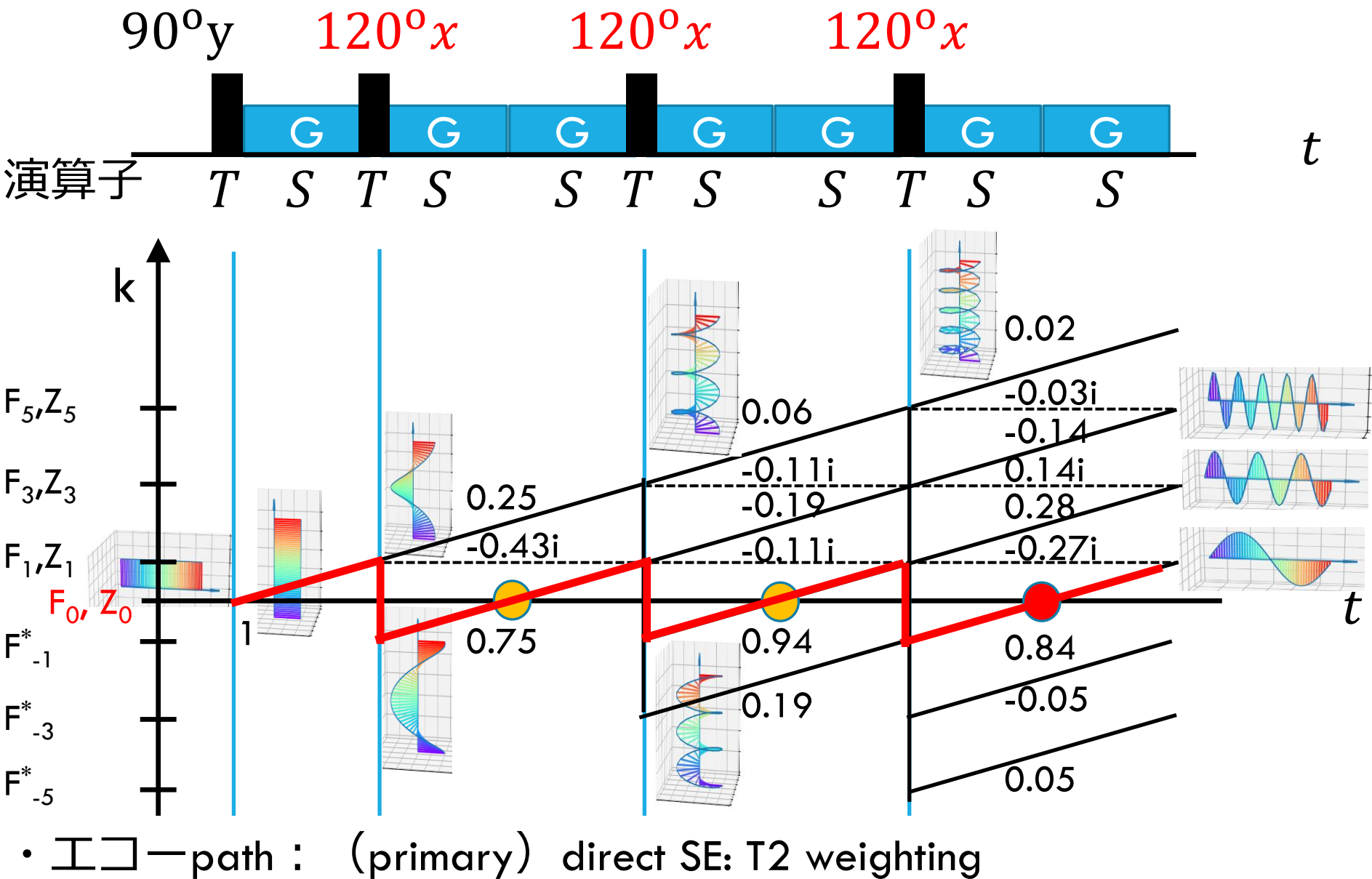
- エコ—強度：行列計算で簡単に求められる

# MULTI ECHO

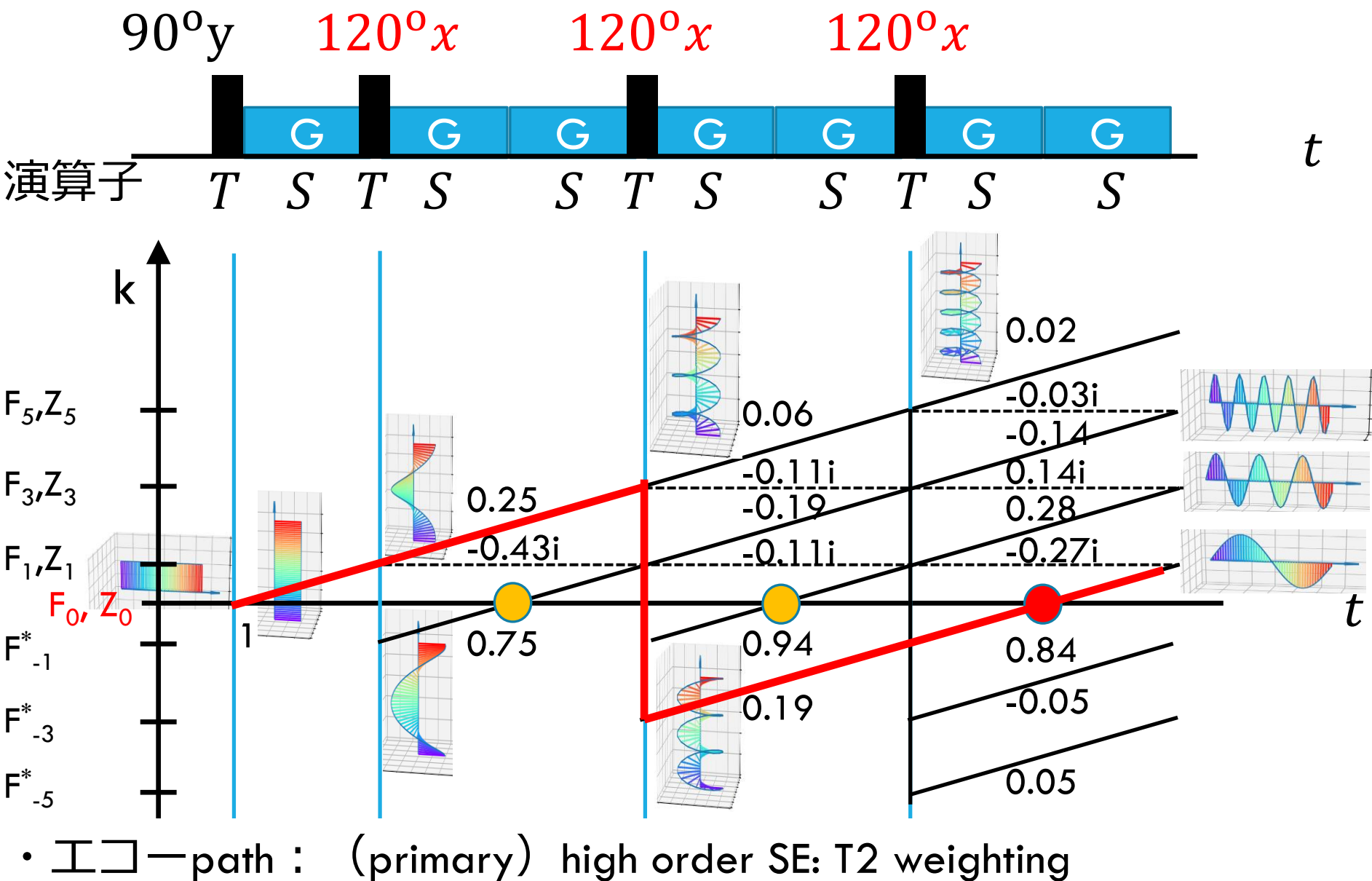


• エコ強度：行列計算で簡単に求められる

# MULTI ECHO

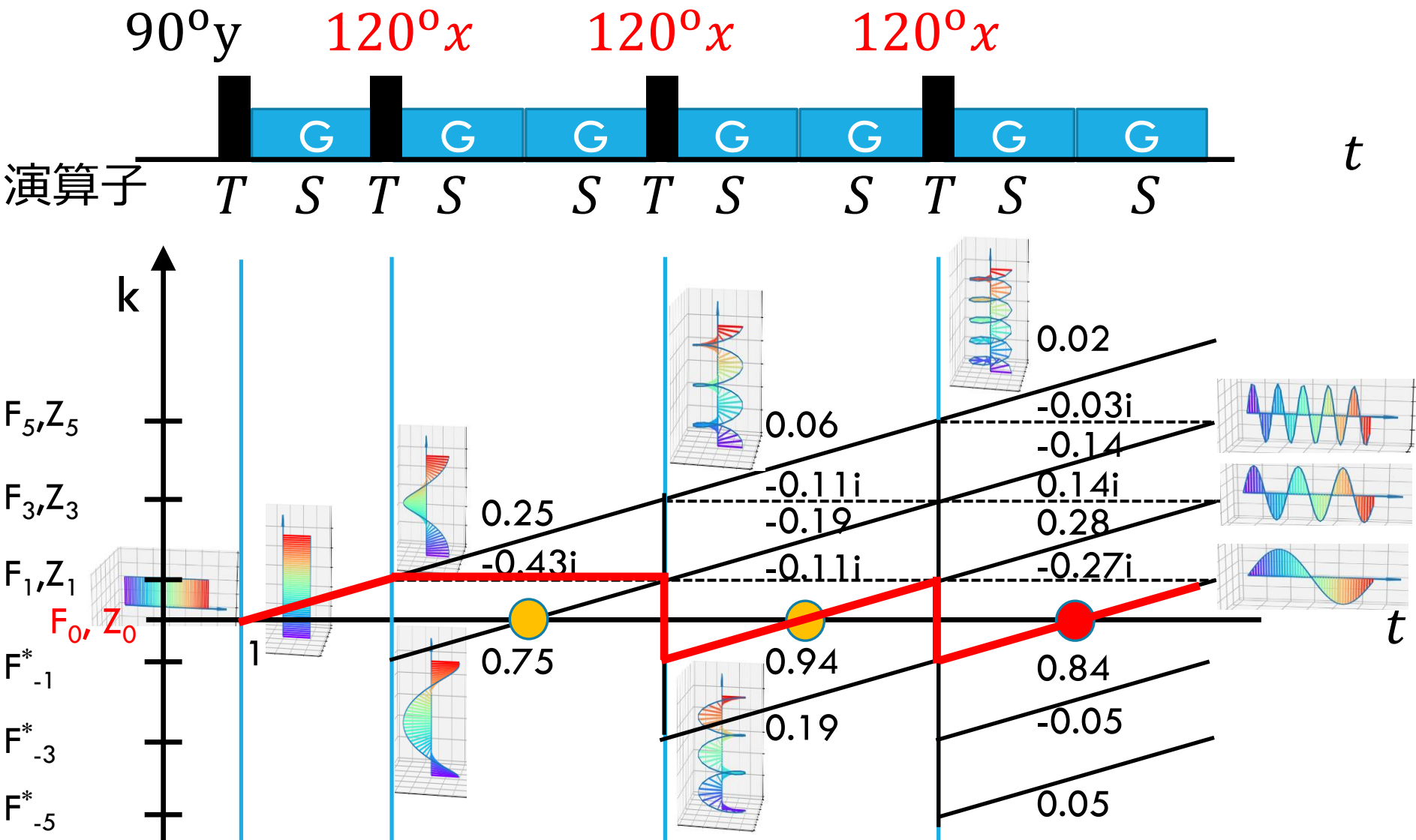


# MULTI ECHO



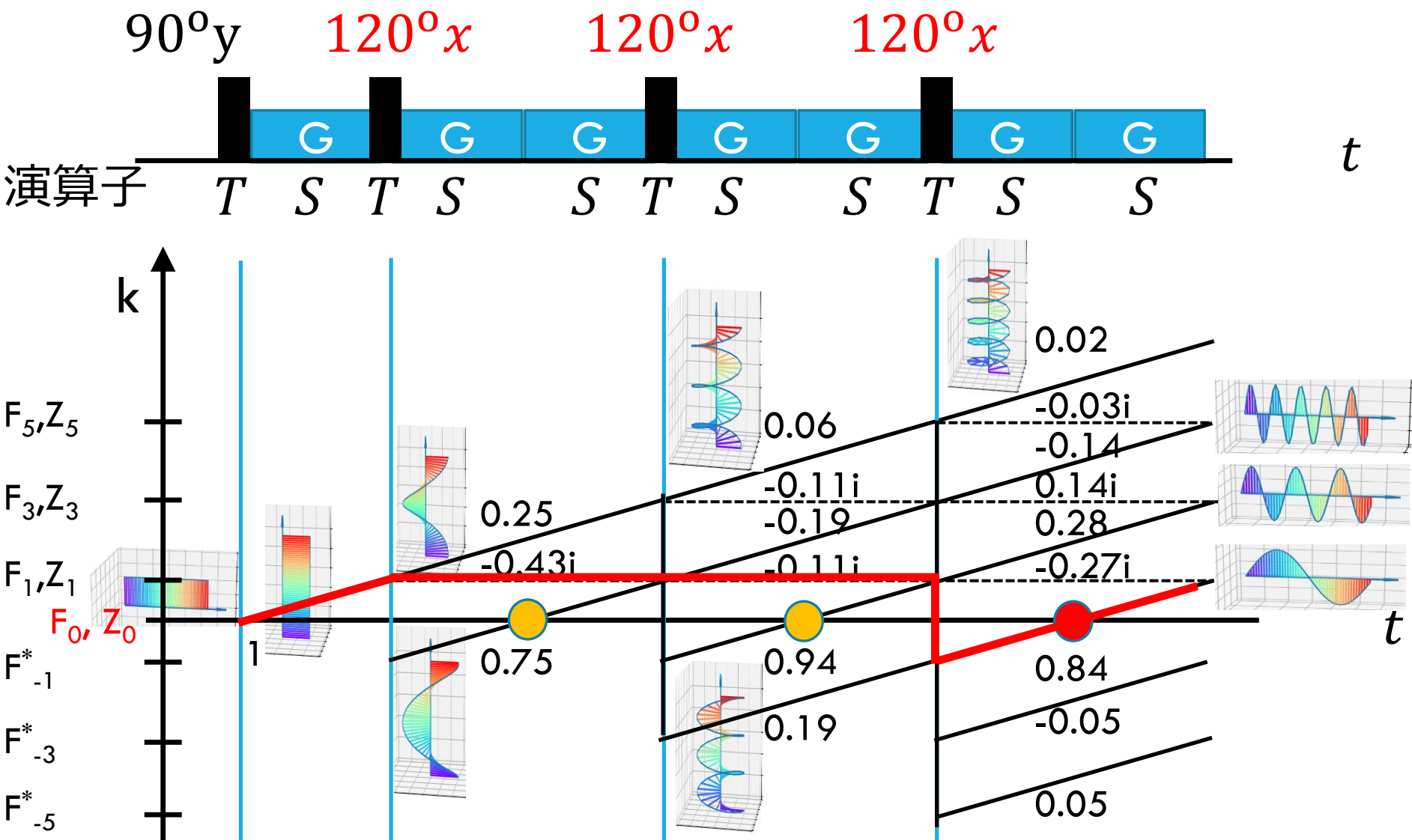


# MULTI ECHO



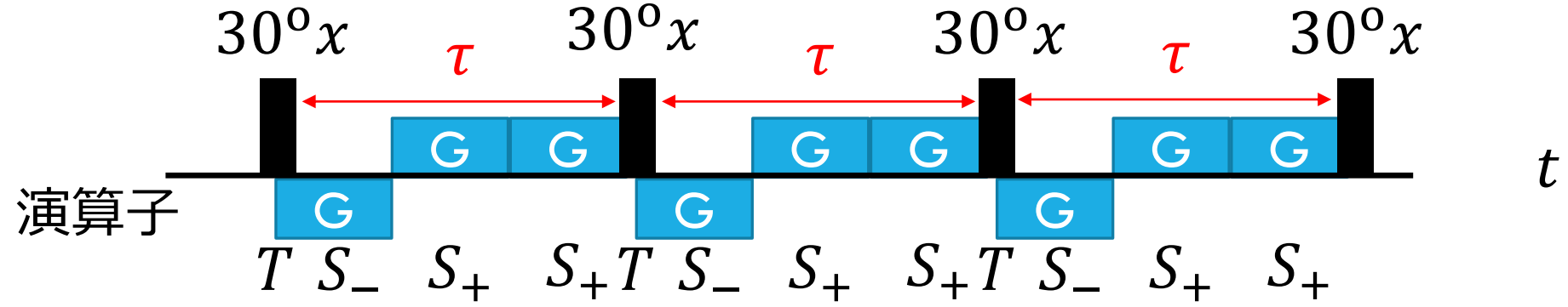
•  $\square$ —path : 1<sup>st</sup> STE + SE : T1 + T2 weighting

# MULTI ECHO



•  $\square$  path : 2<sup>nd</sup> STE + 横磁化 : T1 + T2 weighting

# FAST GRE (FISP/GRASS/FFE/FAST)



$$T_{30x} = \begin{pmatrix} 0.93 & 0.07 & -0.50i \\ 0.07 & 0.93 & 0.50i \\ -0.25i & 0.25i & 0.87 \end{pmatrix}$$

緩和あり  $T_1 = 1s, T_2 = 100ms, TR = 30ms, TE = 20ms$

•  $E$ 演算子をどこに入れるか？

→間に $T$ が入らなければどこにいれてもOK

( $T$ が入ると縦磁化と横磁化が交換するので不可)

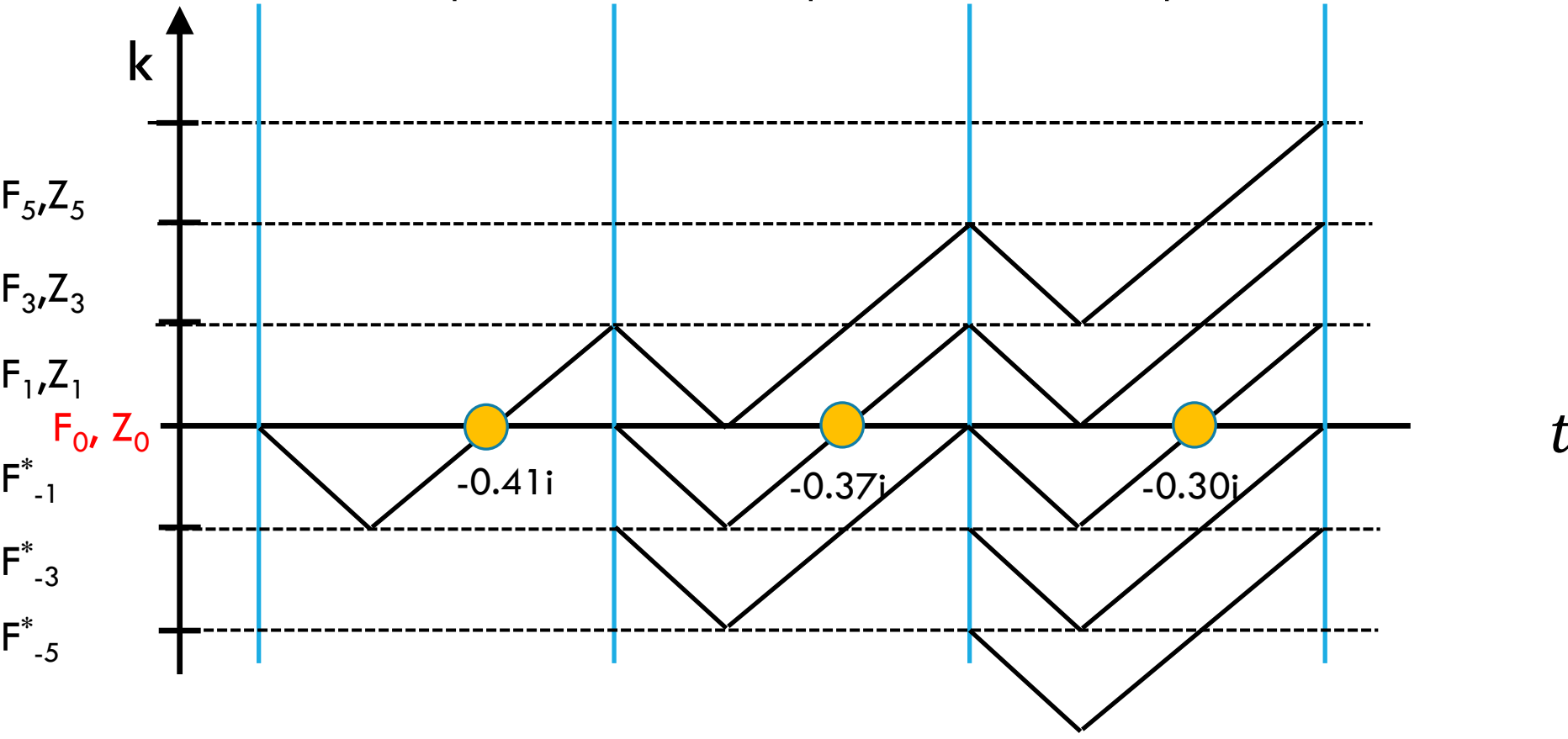
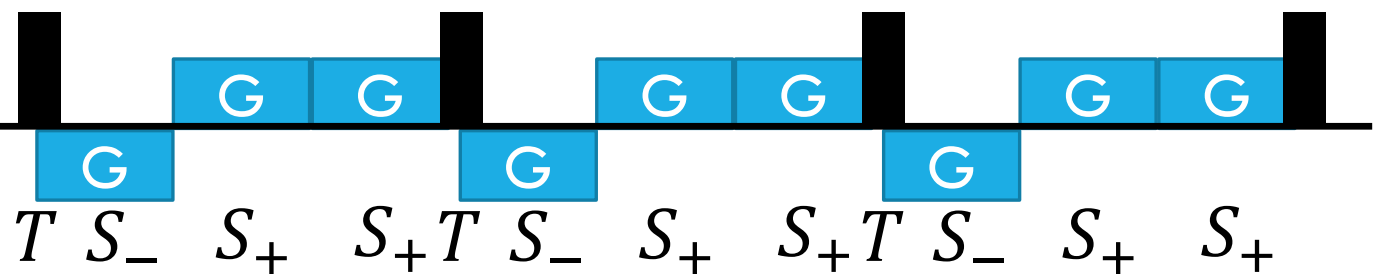
例)  $E(\tau)S_+S_+S_-T = S_+E(\tau)S_+S_-T = S_+S_+E(\tau)S_-T = S_+S_+S_-E(\tau)T$

$$E(\tau)S_+S_+S_-T \neq E(2\tau)(S_+S_+S_-T)(S_+S_+S_-T)$$

# FAST GRE (FISP/GRASS/FFE/FAST)

$30^\circ x$                        $30^\circ x$                        $30^\circ x$                        $30^\circ x$

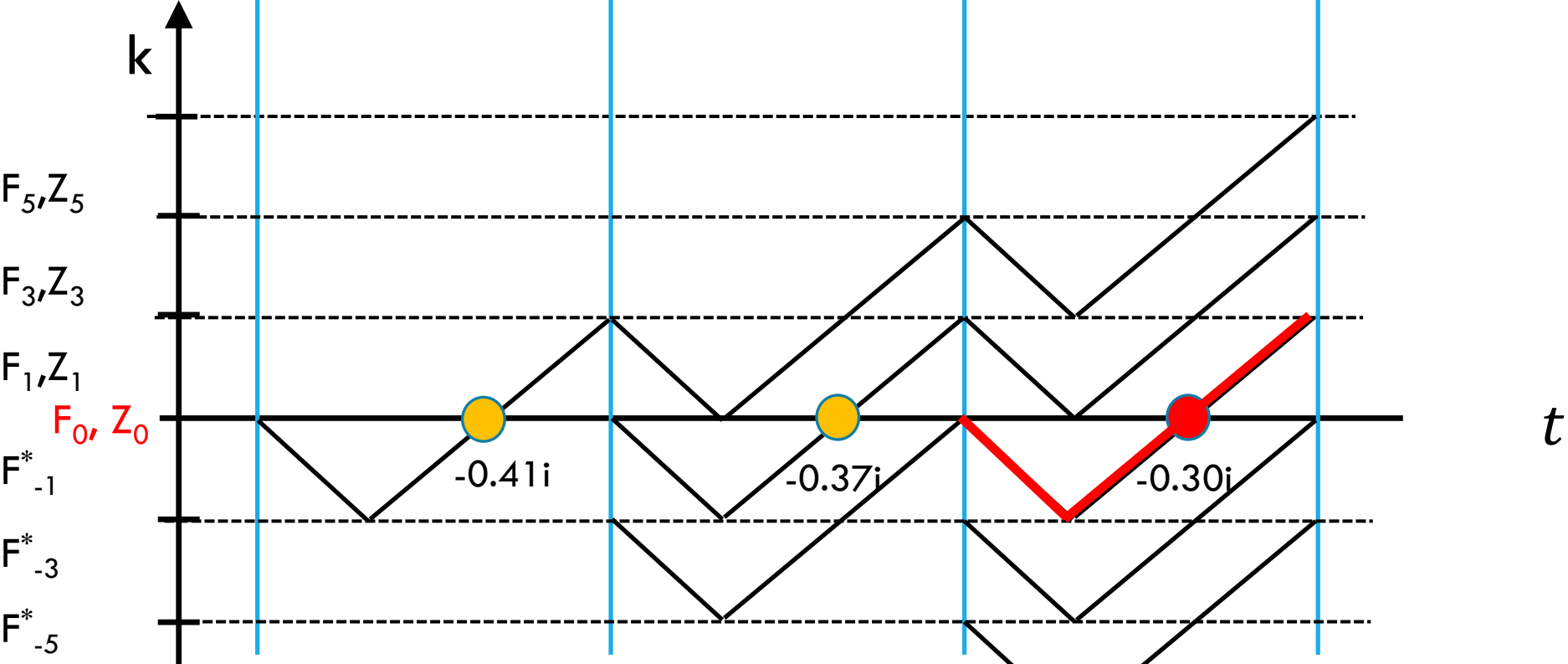
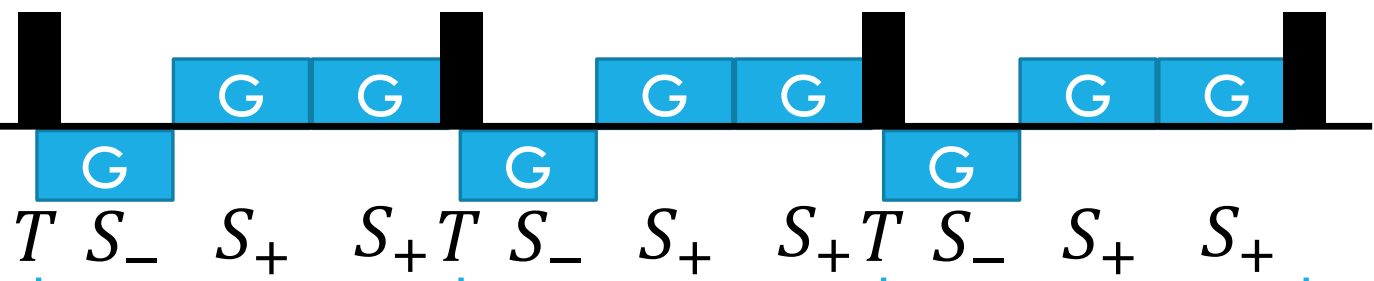
演算子



# FAST GRE (FISP/GRASS/FFE/FAST)

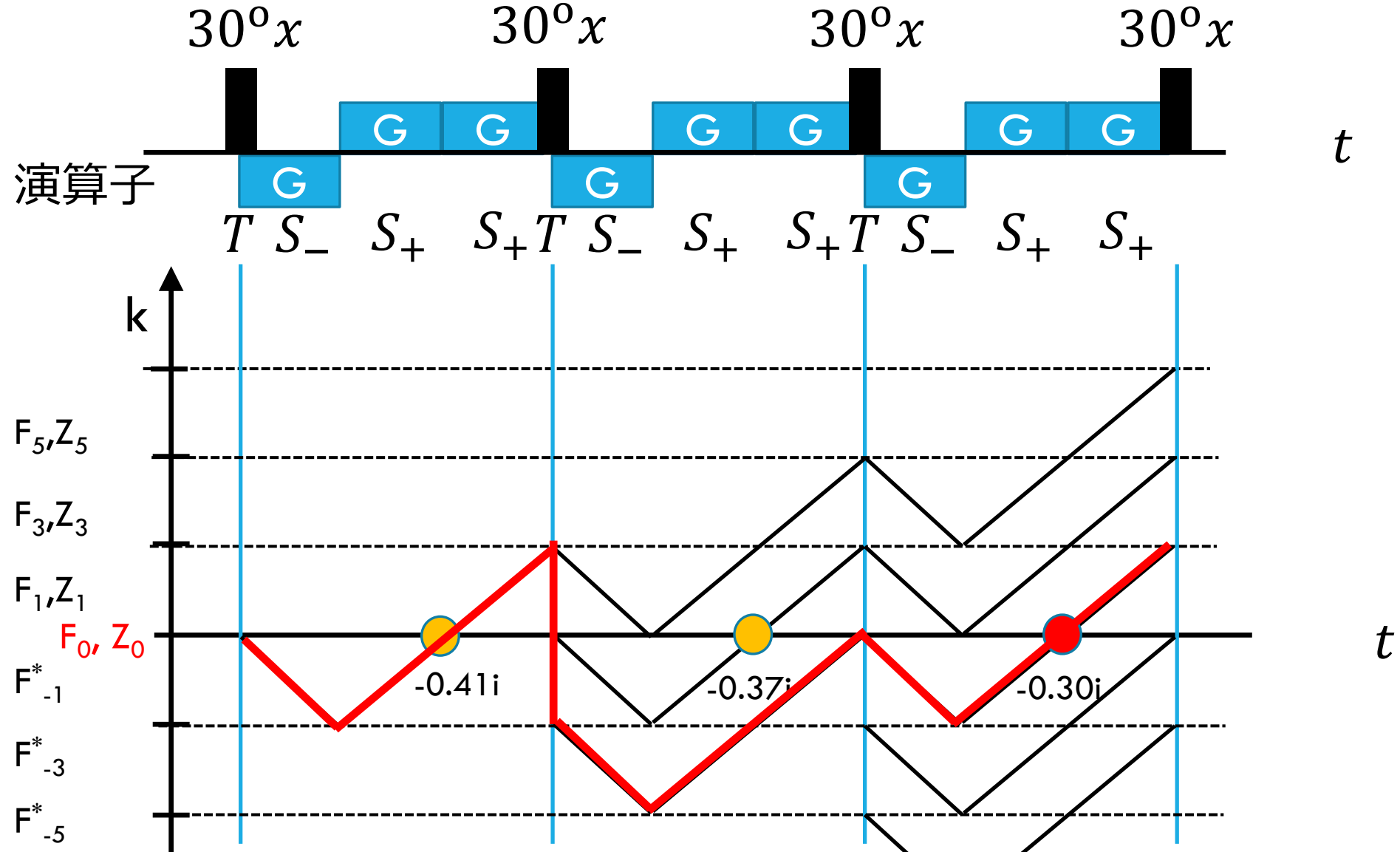
$30^{\circ}x$        $30^{\circ}x$        $30^{\circ}x$        $30^{\circ}x$

演算子



•  $\square$  — path : FID

# FAST GRE (FISP/GRASS/FFE/FAST)



• エコーpath : Gradient spoilingでは横磁化を完全にspoilできない

# これまでのまとめ 2

- $(M_+, M_-, M_z)$ のフリー成分 = “states”  $(F_+, F_-, Z)$
  - statesをまとめて行列 $\Omega$ で表現する
- エコー強度などの計算は行列演算で行う
- RF pulse =  $T$ 演算子
  - Dephasing, rephrasing =  $S(+1), S(-1)$ 演算子
  - 緩和 =  $E$ 演算子
- エコーのpathや組成を知りたいときはグラフが有効
- EPG : 縦軸は $k$  or “states”

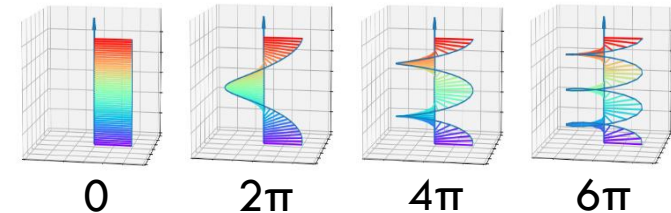
# kの値について

- エコー強度などの計算は行列演算で行う

- RF pulse =  $T$ 演算子
- Dephasing, rephrasing =  $S(+1), S(-1)$ 演算子
- 緩和 =  $E$ 演算子

$k$ の値  
には無関係

$k = \frac{2\pi \times \text{整数}}{\text{voxel size}}$  と定義したが...



実は、これまでの話だけだと、  
エコー強度や $\Omega$ は、 $k$ の値に無関係である

つまり、ボクセルサイズや、Gradientの大きさに無関係



# kの値について

## ■ エコー強度などの計算は行列演算で行う

- RF pulse =  $T$ 演算子
- Dephasing, rephrasing =  $S(+1), S(-1)$ 演算子
- 緩和 =  $E$ 演算子
- 拡散
- 流れ

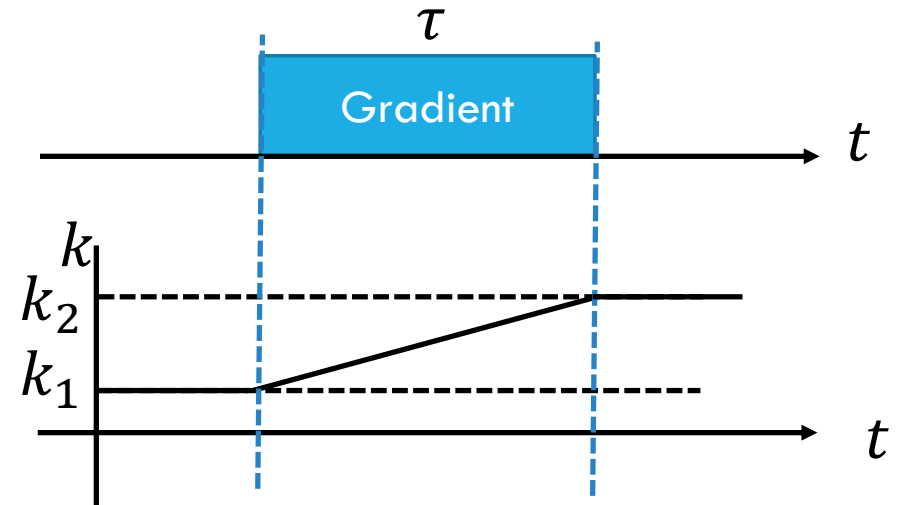
kの値  
には無関係

kの値に依存する

プロトンが移動する現象

# 自由拡散

等方拡散の場合



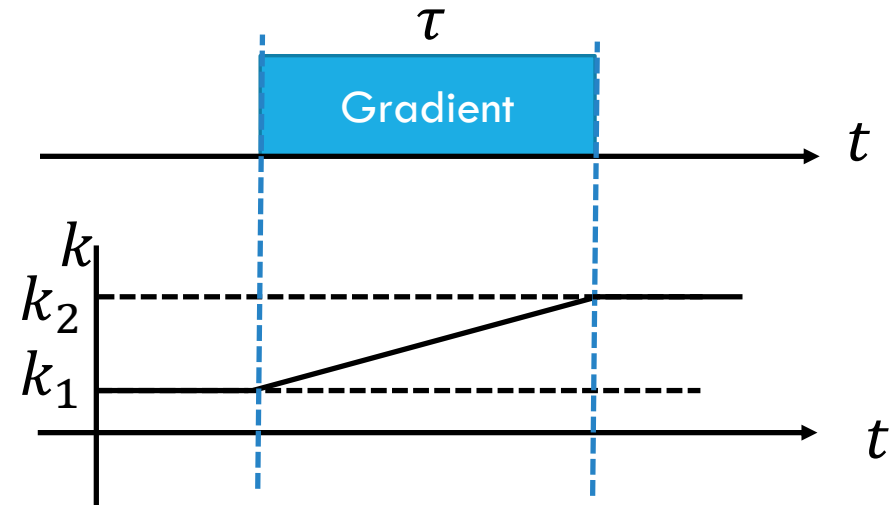
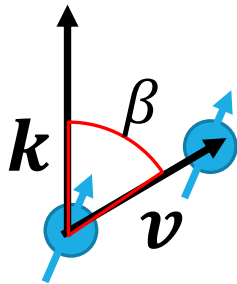
$$D(\tau) = \begin{pmatrix} \exp(-b_{\tau}^T D) & 0 & 0 \\ 0 & \exp(-b_{\tau}^T D) & 0 \\ 0 & 0 & \exp(-b_{\tau}^L D) \end{pmatrix}$$

$$\left. \begin{aligned} b_{\tau}^T &= (k_1 + k_2)^2 \frac{\tau}{4} + (k_1 - k_2)^2 \frac{\tau}{4} \\ b_{\tau}^L &= k_1^2 \tau \end{aligned} \right\} k \text{ の値に依存する}$$

正確な $k$ の値を使ってEPG計算を行う必要あり

## FLOW

G direction



$$J(\tau) = \begin{pmatrix} \exp(-i\alpha_{\tau}^T v) & 0 & 0 \\ 0 & \exp(-i\alpha_{\tau}^T v) & 0 \\ 0 & 0 & \exp(-i\alpha_{\tau}^T v) \end{pmatrix}$$

$$\alpha_{\tau}^T = \frac{k_1 + k_2}{2} \tau \cos \beta$$

$$\alpha_{\tau}^L = k_1 \tau \cos \beta$$

**$k$ の値に依存する**

正確な $k$ の値を使ってEPG計算を行う必要あり

# 連続kへの拡張

$$M_+(\ell) = M_x + iM_y = \int_V F_+(k) e^{ik\ell} d\ell$$

$$M_-(\ell) = M_x - iM_y = \int_V F_-(k) e^{ik\ell} d\ell = \int_V (F_+(-k))^* e^{ik\ell} d\ell$$

$$F_-(k) = (F_+(-k))^*$$

$$\begin{pmatrix} F_+(k) \\ (F_+(-k))^* \\ Z(k) \end{pmatrix}^+ = T \begin{pmatrix} F_+(k) \\ (F_+(-k))^* \\ Z(k) \end{pmatrix}^-$$

または

$$\begin{pmatrix} F_+(k) \\ F_-(k) \\ Z(k) \end{pmatrix}^+ = T \begin{pmatrix} F_+(k) \\ F_-(k) \\ Z(k) \end{pmatrix}^-$$

$T$ は離散kの場合と同じ

# PYTHON CODING

<https://github.com/mrlab-tsukuba/mri-master/tree/master/EPG>

EPG class

```
class EPG:
    def __init__(self, NumOfColumns, m0=1, t1=0.5, t2=0.1):
        self.NumOfColumns = NumOfColumns #CS matrixの行数
        self.setParams(m0, t1, t2)
```

RFパルス演算

```
def op_T(self, Tmatrix):
    self.CSm = self.CS.copy()
    self.CS = np.dot(Tmatrix, self.CSm)
```

シフト演算

```
def op_S1(self): #shift operator ( $\Delta k=1$ )
    self.CSm = self.CS.copy()
    N = self.NumOfColumns
    #check
    if self.CS[0][N-1] != 0:
        print('number of colume for CS matrix is not sufficient!!!')
    self.CS[1][N-1] = np.complex(0,0)
    for i in range(N-1):
        self.CS[1][i] = self.CSm[1][i+1]
    self.CS[0][0] = np.conj(self.CS[1][0])
    for i in range(1,N):
        self.CS[0][i] = self.CSm[0][i-1]
```

# PYTHON CODING

<https://github.com/mrlab-tsukuba/mri-master/tree/master/EPG>

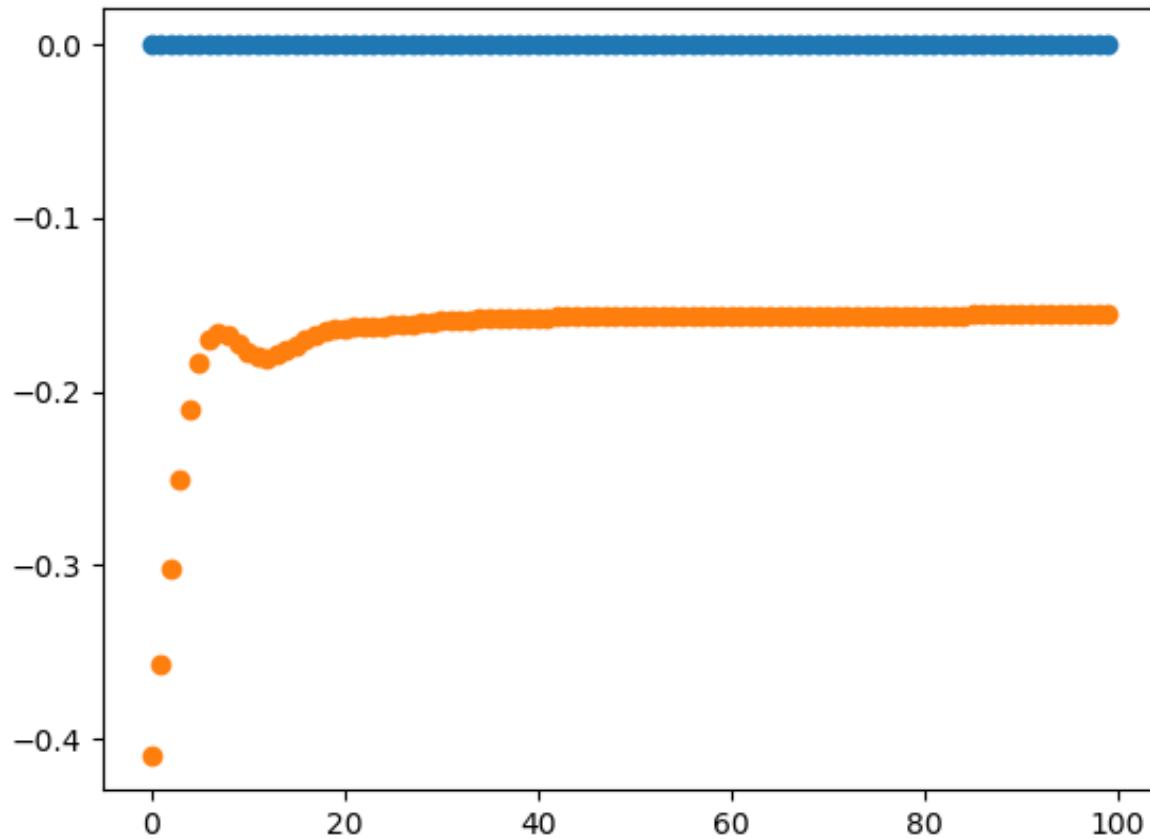
```
def FISP():
    m0 = 1
    t1=0.5
    t2=0.1
    ETL=100
    FA=30
    TR=30e-3
    epg = EPG(ETL+1)
    epg.setParams(m0, t1, t2)
    RF1 = epg.gen_Tmatrix(0, FA/180*np.pi) #90y
    epg.t_echoPeak = np.zeros((ETL), dtype = complex)
    tau = TR/3
    #start
    for i in range(ETL):
        epg.op_T(RF1)
        epg.op_Gx(tau, -1)
        epg.op_Gx(tau, 1)
        epg.t_echoPeak[i] = epg.op_AD()
        epg.op_Gx(tau, 1)
    epg.display_results()
```

# PYTHON CODING

<https://github.com/mrlab-tsukuba/mri-master/tree/master/EPG>

実行

FISP()



実行結果

# まとめ

## 1. EPGの定義

- EPGは、RFパルスの働きを記述する古典的位相グラフに、核磁化分布のフーリエ変換の概念を加えたものである

## 2. $k$ の導入

- 横磁化分布の螺旋のねじれ具合を $k$ で定量化
- dephasingとrephasingを $k$ の増減で表現

## 3. “states”の導入

- 核磁化分布のフーリエ成分 $F_k, F_{-k}^*, Z_k = \text{“states”}$
- “states”をまとめて行列 $\Omega$ で管理

## 4. “states” $\Omega$ の経時変化の計算

- RF pulse  $T$ 、Gradient  $S$ 、緩和 $E$ 、拡散 $D$ 、流れ $J \rightarrow$ 行列演算子

## 5. グラフ

- 縦軸は $k$  or “states”
- エコーのpathや組成

## 6. 実例、python code